

Suppose we have a rigid body, and it's rotating about a fixed axis, and we know that the angular acceleration, α , is given.

Now, it may be a function of time.

And we'd like to find from the angular velocity and how much angle has rotated in some time interval.

So the first thing we have to do is choose a point in the rigid body, and introduce a coordinate system.

Here we have our angle θ .

And with this point, we have our \hat{r} and our $\hat{\theta}$ direction.

And recall that by the right-hand rule, the \hat{k} is going out of the plane of the figure.

And so we have our coordinate system.

And we wrote α as the second derivative of θ with respect to time \hat{k} .

Now our goal is to find ω , which we'll write as the first derivative of $d\theta/dt \hat{k}$, and also to find θ as a function of time.

Now recall that our notation was that α_z was the $\omega_z dt$ equals $d^2\theta/dt^2$.

And in this notation, ω_z was equal to $d\theta/dt$.

Now what we're going to do is we're going to integrate α , just like we did in simple circular motion kinematics for point-like objects.

And so what we have here is that for some time interval, ω_z at time t minus ω_z at time t_0 is the integral of α_z from t_0 to t , which is our component of the angular acceleration.

We have an integration variable, dt' .

And recall that that implies that $\omega_z(t)$ can have some initial value at time $t = t_0$ plus this integral relationship, which is what we want to figure out by direct integration.

Now, this only works when α_z is some function of time.

In order to get the angle, $\theta(t)$, we integrate again, where we have $\theta(t_0)$ is the integral from $t' = t_0$ to $t' = t$ of this function ω_z .

Again, we have some integration variable, t prime, dt prime.

And so we see that θ t can have some initial value plus this integral relationship, t prime equals t of ω z t prime dt prime.

And this is how we can figure out how the point p has a component of angular velocity, and what angle the point p sweeps out in some time interval t .