Let's consider a collision in a lab frame in which one particle is coming in and the other particle is at rest.

So here's particle 2.

V2 initial is 0.

And we could say that m2 is twice m1.

And this is our lab frame.

And now let's consider the same collision in the center of mass frame.

And in that reference frame, we have particle 1 moving with velocity V1 prime initial and particle 2 moving with velocity V2 initial prime.

Now, the way we're going to analyze this problem is that we know the speeds in the center of mass frame relative to the lab frame.

So we actually know these initial speeds.

And we'll write that result down in a moment.

Because we know the initial speeds, we also know that V1 final in the center of mass frame is just minus the initial velocity.

That's the beauty of the center of mass frame.

The velocities just change direction.

They don't change magnitude.

Because we know this we can get V1 final.

And then it's just a simple exercise to go back to the lab frame to calculate the quantity, the velocity of the object 1 in the lab frame.

So that will be our sequence of ideas.

And the key fact that we know is that V1 prime is equal initially to the reduced mass divided by m1 times V1, 2, where V1, 2 initial is just equal to V1 initial minus 0.

So that's V1 initial.

And the ratio, mu over m1, it's very simple to calculate that.

That's just m2 over m1 plus m2, or our case, that's 2/3.

And so from our result, we now have very simply that V1 final prime is minus V1 initial prime.

So it's minus 2/3 V1 initial.

And that's a very straightforward calculation.

We can do exactly the same thing with V2 final prime.

V2 final prime is minus V2 initial prime.

And V2 initial prime is equal to minus mu over m2 times V1, 2 initial.

This ratio instead of being 2/3, it's a simple exercise.

It will be m1 over m1 plus m2 is 1/3 V1 initial.

And so we have solved for the final velocities in the center of mass reference frame.

Now let's just double check our results.

We have V2 final is minus V2 initial.

So that's minus.

But there's another minus sign here.

So we have 2 minus signs.

So that's a plus.

And that's why we have a plus 1/3 V1 initial.

And finally, if we want to ask the question, what are the velocities in the lab frame, it's now a very simple exercise to do reference frame change.

We do need to know what V center of mass is.

That's V1 V initial over m1 plus m2, because remember that V2 initial is 0.

So that's another factor, V1 initial.

And now for conclusion, we have that the final velocity in the lab frame is equal to the velocity in the center of mass frame plus V center of mass.

And we just collect our results, minus 2/3 V1 initial plus 1/3 V1 initial.

So that's minus 1/3 V1 initial.

The outgoing velocity of particle 1 in the lab frame and the outgoing velocity of particle 2 in the lab frame, again, we have 1/3 V1 initial plus another 1/3 V1 initial is equal to 2/3 V1 initial.

So we were able to solve this problem by switching reference frame using our basic fact in the center of mass frame that the speeds remain the same but the direction changes and able to solve this problem without any of the traditional ways of applying the energy momentum relationship and kinetic energy or having quadratic equations.