

MITOCW | MIT8_01F16_L09v02_360p

For a particle that's moving in a circle, we found that when it's moving at a constant rate of $d\theta/dt$ and let's recall what we meant by θ of t and here's our particle, and we introduced our polar coordinates r and θ , then we found that the velocity was $r d\theta/dt$.

And so let's assume that this quantity is positive, in which case the velocity is pointing in the positive θ direction.

And that means that everywhere in the circle, the velocity is tangential to the circle, and the magnitude is a constant.

So for this case of uniform circular motion, we calculated that the acceleration was equal to minus $r d\theta/dt$ squared, which means that at every point, the acceleration vector is pointing towards the center.

Now we can write that acceleration vector as a component a_r of r where this component is given by r times $d\theta/dt$ squared.

It's always negative, because when you square this quantity, it's always a positive quantity.

The minus sign, just to remember-- that means that the acceleration is pointing inward.

Now how can we think about that?

Well, if we look at the velocity vector, what's happening here is the velocity is not changing magnitude but changing direction.

And if you compare two points-- and let's just pick two arbitrary points.

So let's remove this acceleration for a moment and consider two arbitrary points-- say, a time t_1 and t_2 .

So our velocity vectors are tangent.

The length of these vectors are the same.

And if we move them tail to tail-- [v_2 - v_1] and take the difference, Δv , where Δv is equal to v_2 minus v_1 , then we can get an understanding why the acceleration is pointing inward, because recall that acceleration by definition is a limit as Δt goes to 0.

That means as this point approaches that point of the change in velocity over time.

And so when we look at this limit as we shrink down our time interval between t_2 and t_1 , then this vector will point towards the center of the circle.

And that's why the direction of \mathbf{a} is in the minus \hat{r} direction.

Again, let's just recall that this is the case for we called uniform circular motion, which is defined by the condition that $d\theta/dt$ is a constant.