Let's consider another example of continuous mass transfer.

Suppose we have a truck, and that truck has some type of plow.

And it's plowing snow.

And there's some type of external force acting on this truck, friction, pushing the truck forward, so let's just assume we have some type of force, F, on the truck.

And this is our snow.

And what's happening in this problem is that the truck connects-- picks up the snow.

And then, which is at rest initially, gets the snow up to the speed of the truck.

And then the snow falls off the plow.

So how do we model this problem?

Well, let's look at our situation at time t.

And what we're going to do is, we're going to consider a certain mass of snow, delta ms, that's at rest.

And our truck, it's a fixed mass truck, is moving with a velocity vt at time t, the truck.

So now, what happens at time t plus delta t?

Well, the truck has picked up the mass of the snow.

And the truck has now changed its speed.

So that's at time t plus delta t.

And now we want to write down our momentum law.

So we have our external force.

Let's call this the plus i hat direction.

We have our external force is equal to the limit as delta t goes to 0 of the momentum at time t.

So we have t plus delta t, so what we have is the mass of the truck plus delta ms times v of t plus delta t.

And we have to subtract from it.

That's divided by delta t.

And we have to subtract from this the moment at time t.

The snow is at rest, only the truck is moving, so we have minus t of vt divided by delta t.

Now, as usual, we're going to say that the truck has changed its velocity in this interval.

And what we want to do now is write out our equation.

We have F equals the limit as delta t goes to 0.

Now when we write this out, notice that we're going to have a number of terms here.

We have mt plus delta ms times v plus delta v of the truce-- that's the first term-- divided by delta t.

And the second term is just minus mass of the truck, vt over delta t.

Well, first off, we see some cancellations between this term, this, and that.

And what we're left with is limit as delta t goes to 0.

We have the snow term, delta ms over delta t times the truck.

Now, here we have a term, which we're going to analyze in a moment.

It has two infinitesimal quantities.

And this term is of second order, which we're going to neglect.

And finally, we have the term of mass of the truck times delta vt delta t.

So, neglecting this second-order term in differentials, what we get when we take the limit is, that we get the force is the rate that the snow is being picked up times v of the truck, plus mass of the truck, times v truck, dt.

Now, the only issue that we have to think about here is about the rate that the truck, the snow, is being picked up.

So that's our last consideration, but this will be our differential equation for adding mass, continually, to a system.

So, when we found our equations for describing the rate that the truck changes its speed-- the snowplow when it's pushing snow away, we wrote down our equation in terms of the external force on the truck, the rate that snow is being picked up by the truck, the velocity of the truck, the mass of the truck, the rate of change of velocity of the truck.

If we multiply our equation through by dt, we have the following equation.

And now what we want to consider is we want to focus on how much dms is picked up in our infinitesimal time, dt.

So one way to think about that is, let's do a little drawing.

So here's our truck.

And let's say at time t-- the plow is right here, so here's the picture at time t.

And let's draw some snow here.

Here in New England we have lots of snow.

And at time t plus delta t, the truck has moved a certain distance.

And so, this quantity is the amount of snow that's picked up by the truck and displaced.

Now, the truck has moved a distance v truck delta t.

And so, we can identify the delta ms, which in the limit will be dms, is equal to the density of snow times the cross sectional area of the plow, times the length v of truck delta t.

And so now we have an expression for the rate that the mass is being picked up in this time interval.

And we can write that in the following way.

That we see that our differential equation Fdt equals dms, which is rho A vt, times dt-- This will be a small interval, we're taking a limit now-- times another vt plus mt dvt.

Now, we can bring this term over to the other side.

And so we have F minus rho A vt squared times dt equals the mass of the truck times dvt.

And the mass of the truck, in this case, is fixed.

Why?

Because when the snow gets into the truck, at the end of this delta t time interval, it gets displaced to the side.

So this is an equation that we can separate and integrate.

So we have dt is equal to mt divided by F minus rho A vt squared dvt, and we integrate from some initial time and some final time.

And here, we're integrating from v of truck from some t initial time to some final time.

And that's our integral version for our-- finding the velocity.

This interval is an interval that's not hard to do, that you should try as an exercise in elementary calculus.