Let's consider a one dimensional motion that has a non-uniform acceleration.

What we'd like to do is explore how do you differentiate position functions, to get velocity functions, to get acceleration functions.

So what we're going to consider is a rocket.

So I'm going to choose a coordinate system y.

And here's my rocket.

And I have a function y of t.

And I'll have a j-hat direction, but this will be a one dimensional motion.

Now I want to express while the rocket is thrusting upwards and the engine is burning, we can describe a function y of t to be equal to 1/2 a constant a naught minus the gravitational acceleration, times t squared.

And we're going to have a separate term here, which is minus 1 over 30.

And you'll see where this 30 comes in as we start to differentiate.

The same constant a naught, t to the 1/6 over t naught to the 1/4.

Now in this expression, a naught is bigger than g.

And also, this is only true, this holds for the time interval 0 less than or equal to t, less than t naught.

And at time t equals to t naught, the engine shuts off.

And at that moment, our expectation will be that the y component of the acceleration should just be minus g, for t greater than t naught.

So now let's calculate the acceleration as the velocity and the position as functions of time.

So the velocity-- in each case, we're going to use the polynomial rule.

So the y component of the velocity is just the derivative of t squared, which is just 2t.

And so we get a naught minus g times t.

And when we differentiate t at the 1/6, the 6 over 30 gives us factor 1 over 5.

So we have minus 1 over 5 times a naught, t to the 1/5 over t naught to the 1/4.

And this is a combination of a linear term and a term that is decreasing by this t to the 1/5 factor.

And finally, we now take the next derivative, ay of t, which is d dy dt.

I'll just keep functions of t, but we don't really need that.

And when we differentiate here, we get a naught minus g.

Now you see the 5s are canceling, and we have minus a naught t to the 1/4 over t naught to the 1/4.

Now at time t equals t naught, what do we have?

Well, ay at t equals t naught.

This is just a factor minus a naught.

Those cancel, and we get minus g, which is what we expected.

Now this is a complicated motion.

And let's see if we can make a graphical analysis of this motion.

So let's plot y as a function of t.

Now notice we have a quadratic term and a factor t to the 1/6 with the minus sign.

So for small values of t, the quadratic term will dominate.

But as t gets larger, then the t to of the 1/5 term will dominate.

That's t squared.

And let's call t equal to t naught.

Now we have to be a little bit careful.

Because when the engine turns off, the rocket is still moving upwards.

So even though it starts to grow like this, it will start to still fall off a little bit, due to this t to the 1/6 term.

It has a slope that is always positive.

So we're claiming that our velocity term is positive.

And then somewhere, if the engine completely didn't turn off, this term would still-- where is the point where the velocity, the vertical velocity is 0, because gravity will-- this term will eventually dominate.

And that, we can see, is going to occur at some later time, even though that's not part of our problem.

Now in fact, if we want to define, just to double check that, where the y of t equals 0, then we have a naught minus g over t equals 1/5 a naught t to the 1/5 over t naught to the 1/4.

And so we have the quadratic-- we have this equation, a naught minus g times t naught to the 1/4 equals t to the 1/4.

Or t equals 5 times a naught minus g, a quantity bigger than 1, times t naught.

This quantity is larger than 1.

And so we see that when given this motion, the place where the velocity reaches-- the position reaches its maximum would occur after t equals t naught.

So this graph looks reasonable.

And that would be the plot of the position function of the rocket as a function of time.

As an exercise, you may want to plot the velocity as a function of t 2, to see how that looks.

That would correspond to making a plot of the slope of the position function.