We will now like to explore a property of work done by forces called path independence.

So recall that our work is defined to be an integral of F.ds from some initial to some final point.

Let's look at an example.

The force that we're going to look at is the gravitational force.

And let's draw a coordinate system.

Let's take an initial point and a final point, and let's choose plus x and plus y and our unit vectors i-hat and j-hat.

And in this example we're going to consider the gravitational force near the surface of the earth, which is a constant pointing downwards.

And what I'd like to do is consider two paths.

I'd like to consider a path, that first path goes straight up and over.

So this will be path 1.

It has two legs.

And the second path, path 2 will go horizontal and vertical.

So that's going to be path 2, and it has two different legs.

And we'd like to evaluate this integral on both paths.

Now the way we do that is, first off, let's start with path 1.

We'll call this y initial and we'll call that y final.

And we'll draw the gravitational force mg, mg on both legs.

And now we see that when we do the integral along this first leg of path 1, the gravitational force is opposite the direction we're moving the displacement.

So the interval being negative, the force is constant.

And so the work done is simply minus mg times delta y, which is y final minus y initial on that first part.

Now on the second part, this is a right angle.

And because it's a right angle the dot product is 0.

So that's the total work done on path 1.

Now on path 2, we'll again, let's just draw our forces.

We have gravity down and we have gravity down.

And we can see that, once again, on the first horizontal leg of path 2 this integral is 0 because it's moving perpendicular to the force, the direction of just along the path.

And on this leg, just as on that leg there, the gravitational force is down, the integral is negative, we get exactly the same result.

Now what we'd like to do is consider a more general path rather than these two horizontal and vertical legs.

So let's draw a coordinate system again.

Let's introduce i, and let's introduce f plus y and plus x.

And now let's consider a path which is moving like that.

Now on this path, what we want to do is break it down into horizontal and vertical pieces, horizontal, vertical, horizontal, vertical, horizontal, vertical, horizontal, vertical, horizontal, vertical.

And let's focus on one of these pieces, which has a displacement, ds, going from one point to another.

Now if we blow that up, so we have our displacement, ds, then what we have here is a displacement, dx, in the horizontal direction and dy in the vertical direction.

And so we see again that when we compute this work, and our gravitational force is downward, then the horizontal display-- part of this displacement, the dot product in the horizontal direction, 0, only the vertical part cancels.

And so when we add up the work along these horizontal and vertical pieces, only the vertical piece counts.

We're just adding up the dy's until we get y final minus y initial.

Now let's be a little more analytic here and write out our ds as a vector dx i-hat plus dy j-hat and our gravitational force as minus mg j-hat.

Now the work is the integral from the initial to the final place of fg.ds.

And so when we do that we have minus mg j-hat dot dx i-hat plus dy j-hat.

And from her initial to the final place.

And notice that we have j-hat dot i-hat.

They are perpendicular so that dot product is 0.

And j-hat dot j-hat is 1.

And so what we're left with here in the integral is simply now we're integrating from y initial to y final because we only have minus mgdy.

And that's what we said before, only the vertical parts are adding up.

And so when you do this integral, mg is a constant.

We get mg times y final minus y initial, and that agrees with what we had before.

So this type of force is an illustration of a force where the work done is independent of the path we choose from the initial to the final place.