Let's consider the motion of a wheel that's rolling along the ground with some center of mass velocity vcm.

And because the wheel is rotating it has an angular velocity.

And you can see that that vector is directed into the plane of the board.

Now, what we'd like to do is consider the kinetic energy of this continuous body.

A little bit later on, that body has moved some distance.

And what we want to consider is the fact that not only is every point in the body moving with the center of mass speed, but there's this additional rotational energy that's associated with the fact that every point in the center of mass reference frame is undergoing circular motion.

So how do we describe that?

Well, we'll do that by choosing some point in the body.

So let's pick a point.

We'll call that mj, with mass mj.

And the velocity of this point, remember, has two components.

To simplify it, we'll give ourselves a little more picture here.

Every single point in the object has the vcm.

But because this object is undergoing circular motion, there is vcmj.

That's the rotational circular tangential velocity.

And so the vector sum of these two is the actual velocity vj of the j-th object.

vj is equal to the center of mass velocity plus the tangential rotational velocity that it has, because it's undergoing circular motion.

And now what we'd like to do is calculate the kinetic energy of this object.

Well, the kinetic energy is the sum j from 1 to n of 1/2 mj times the velocity of this j-th particle squared, which we can take as a dot product.

So we can write that as vcm plus vcmj dot vcm plus vcmj.

And that's just vj squared.

So when we look at these terms, it looks complicated at first.

But there's some nice-- there's going to be vcm dot vcm.

There's two cross terms.

They're identical.

And vcmj dot vcmj.

So let's write out those three terms.

We have 1/2 mj.

vcm dot vcm is vcm squared.

Now, every point in the object has the same vcm.

So we can pull that one out of the sum.

And now we'll take these cross terms.

So we have the sum over j from 1 to n.

There's two cross terms.

So the 2's are going to cancel.

And inside here, we have to remember to keep our mass element.

That's important.

Now, I'm going to write it as mj vcmj vector.

Now, remember, when you dot with vcm, every single point has the same vcm.

Every j-th element has the same vm, so I can pull that vcm outside.

And finally, I have the last term, which is the sum over j from 1 to n of 1/2 mj vcmj squared.

And that's just the dot product of those two terms.

And so our kinetic energy looks rather complicated, but let's focus on this term right here.

Because recall from our video on the center of mass that the definition of the center of mass reference frame, so if you're moving in the center of mass, that in the center of mass reference frame, the sum of mj vcmj is equal to 0.

So for instance, if you're in the center of mass frame, you're moving with vcm.

The only velocity is this.

And in that frame, the sum of mj vcmj is 0.

And we did a video on that one before.

And that's exactly what's in this term.

So this term is 0.

So this, remember, was how we defined the center of mass reference frame.

And therefore, our kinetic energy consists of two pieces.

This first piece is just 1/2 the total mass times vcm squared.

And our second piece over here, we'll just write it out now-- 1/2 sum over j mj vcmj squared.

Now, if you are moving with the center of mass, then this j-th object is just undergoing circular motion.

And so we have our result that we've used many times is that the velocity, the tangential rotational velocity, is just equal to the radius rsj-- so let's introduce that rsj-- times the angular speed omega.

And when we put that into this term, we see our kinetic energy has two pieces-- m total v center of mass squared plus 1/2 j goes from 1 to n-- I didn't finish that sum there-- mj rsj squared.

Now, just remember that every single point in the object has the same angular speed, and so we can pull out the omega squared in there.

And because this is a continuous body and we take the limit, as we've done before, as mj goes to 0, this quantity of mass times distance squared is just the moment of inertia about the center of mass of that body.

And in conclusion, K is 1/2 m total v center of mass squared plus 1/2 the moment of inertia about the center of mass times the angular speed squared.

Now, this is the same crucial decomposition that we've talked about many times.

This first piece is what we call the translational kinetic energy, because it just represents how the center of mass is moving.

And the second piece is what we call the rotational kinetic energy, because it's a representation of just the kinetic energy of rotation.

For example, if you were in the center of mass frame, there would be no translational energy, and this would be the only kinetic energy.