So when we analyzed Newton's Second Law-- applied to this compound system-- we had two equations for Object 1 and Object 2.

And what we found is that we had three unknowns-- the tension in the string and the accelerations of the two objects.

Now, how do we solve this system?

Well, we're missing one condition, which is a constraint condition.

Which is, as Object 2 moves and Object 1 moves, they have to move in some relationship together.

And now what we'd like to do is show an analytic approach for finding that constraint condition.

And the way we think about it is that, we'll call l equal to the length of the string, and this quantity is a constant.

And what we'd like to do is introduce coordinate functions for our Object 1 and our Object 2 and express l in terms of those coordinate functions.

And then, take two derivatives of l, set that equal to 0 because it's a constant, and that will give us a relationship between the accelerations of Objects 1 and 2.

Now, this is a little bit tricky.

And so, what we want to do is, very carefully, show how we introduce coordinate functions.

Recall that we had j hat 1 down and j hat 2 downwards.

What that implies is that we're choosing some origin and we're-- let's choose an origin up here-- and for a coordinate function for Object 1, it has to be consistent with our choice of what we mean by j hat 1.

So, in this sense, y1 is a positive quantity when we're going downward.

Now, what about coordinate functions for the other objects in the system?

Well, let's look at a few things first.

This is a fixed distance-- we'll call it s1-- between the ceiling and the center of the pulley.

And let's make each of the pulleys a radius, r.

And let's call this a function y b.

And let's make this y2 of t.

So now we have coordinate functions for 2 and coordinate functions for 1.

And again, recall that this distance here-- s2-- is a fixed distance.

And when we define these coordinate functions in this fashion, we know that the second derivative-- d squared y1 dt squared-- this is precisely what we mean by the acceleration of Object 1.

And in the similar fashion, d squared y2 dt squared is what we mean by the acceleration of Object 2.

So we've introduced a coordinate system, we've made it very clear what we mean by the accelerations of a1 and a2, and now let's look at our constraint condition that the length of the string is constant.

So what we're going to do is try to see if we can express the length of this string in terms of all the coordinate functions and some of these ancillary quantities.

So what we have here is that the length of the string is y b going down here.

So the length of the string has a factor y b.

It wraps around Pulley b-- so that's pi R-- and it goes up to this length here.

Now, this length is y b minus s1.

So that's y b minus s1.

We wrap around the pulley again-- that's pi R.

And now, we have this length here, which is y1 minus s1.

s1?

Yeah.

And let's just make sure we have all of our quantities here.

Now, we also have another constraint condition, that this length of the string-- we're going to call this l1-- we have a second string here, l2.

And l2 is given by y2-- this length is y2 minus y b-- and that was what we called this constant, s2.

So we now have two string lengths-- l2 equals y2 minus y b-- and both of these string lengths are constant.

And we have the following facts.

Let's start with this one first, that the second derivative of l2 dt squared-- because the length of the string is a constant, that's 0.

And that tells us, two derivatives of this is a2 and two derivatives of that is a b.

And this is something that we saw before, that block b and 2 are moving together.

So when we treated the system as just b and 2 together, we see that the acceleration of 2 and the acceleration of Pulley b are the same.

So we could have just said that before we began.

Now, let's put these equations aside for the moment.

And now let's consider taking two derivatives of String 1.

Recall, String 1 is this object here.

We'll call this String 1 and this String 2.

Now, again, if we take two derivatives-- let's look at our expression first-- we see that we have two factors of y b and we have a bunch of constants whose derivative is 0.

So we don't have to worry about the wrap around distances-- the pi R's-- we don't have to worry about the constants-- s1.

All we have to think about is which quantities are changing in time.

So we have 2 a b when we take two derivatives and we have one factor of a1-- and that's 0.

And now, because the block and 2 and Pulley b are accelerating together, we have our condition, which is 2 a2 plus a1 is 0-- or that a1 is equal to minus 2 a2.

And that is the extra constraint condition that will enable us to solve the system of equations.