

Now we'd like to talk about angular velocity.

So for a particle traveling in a circle, we've seen that the velocity can be written as $r \hat{d}\theta/dt$.

In the $\hat{\theta}$ direction.

And we've also seen that $d\theta/dt$ can be positive, in which case, θ is increasing.

And so the particle is traveling around the circle in this direction.

You can call that the counterclockwise direction.

We've also seen that $d\theta/dt$ can be negative, in which case the angle is decreasing, so the particle's traveling around the circle in this direction, which we could call clockwise.

Let's now look at rotation in an arbitrary plane.

So if I have a plane like this and I have some particle traveling in a circle like this.

And I have some observer that's above the plane looking down on this plane, then it will see this rotation as being in the counterclockwise direction.

Whereas if I have another observer down here and they're looking up at this plane, they'll see the motion as being clockwise.

And so you can see we have a need for the more formal definition for the rotation of this.

And so what we're going to do is use the right hand rule to define a direction that tells you both the direction it defines the plane and it also tells you what the positive direction of rotation is for that plane.

So the way we'll do this right hand rule is we'll take our right hand, we'll curl our fingers in the direction of the rotation.

And our thumb will point in the direction of the positive direction that we're defining.

So in this case, I'm going to have an arrow like this.

I'll call it \hat{n} to indicate that it's the normal unit vector to that plane.

All right.

Let's come back to this example now, where the plane of the motion is this plane of the board.

And let's look at our two cases again.

So in the case of $d\theta/dt$ positive, our circle looks something like this.

And you can see that if I use my right hand rule, the plane, the vector that I've defined, is out of the board.

And I'm going to use this symbol, a circle with a dot, to indicate this direction of out of the board.

In our other case, for $d\theta/dt$ less than zero, our particle is traveling in this direction.

And so you can see by my right hand rule that now the direction that I've defined is into the board.

And for that, I'm going to draw this as an \times in the circle.

So these are symbols that you'll see throughout the rest of the course, the out of the board and into the board symbols.

And so now, let's look back at our coordinate system that we've defined.

We have \hat{r} direction and \hat{a} direction.

And in this coordinate system, there's a third direction that's defined, which is the \hat{k} direction, which you can see is out of the board.

And so now, I'm going to define what we actually call the angular velocity as ω .

And I'm going to write that as $d\theta/dt$.

Now, in the \hat{k} direction, in this case, it could be in an arbitrary \hat{n} direction, depending on what you're playing of motion is.

In this case, I'm going to call it in the \hat{k} direction, and now you can see that these signs are going to line up with these directions.

So ω , as $d\theta/dt \hat{k}$, when $d\theta/dt$ is positive, \hat{k} is in the same direction as this unit normal that we've defined that defines both the plane of the motion and the direction of the motion.

And when $d\theta/dt$ is negative, I have that ω is in the negative \hat{k} direction, which is exactly this direction here that we've defined as being the motion for this plane.

And so this is how we define angular velocity.

So we can define $d\theta/dt$ to be the component of the angular velocity ω_z , because it's in the \hat{k} direction.

And we can also define the angular speed, which is just ω as being the absolute value of this angular velocity ω .

So that is, in other words, it's just the absolute value of the $d\theta/dt$.

So when a particle's undergoing circular motion, it has a velocity that you can describe, which is the tangential motion around the circle.

And it also has an angular velocity, which we define as being in a direction that's perpendicular to that direction of rotation.

And it's defined by the right hand rule.

And now that we have defined this ω_z , the component of the angular velocity, we can rewrite the velocity as just being equal to r times ω_z still in that $\hat{\theta}$ direction.

And so this is another way that we can write the velocity and connect it back to the angular velocity.