Let's consider the motion of a car on a circular track, and the track is frictionless.

And it's also banked.

So this is the overhead view of our circular track.

It has radius, r.

And here's our car moving at a constant velocity.

Now, from the side view when we want to look at that bank turn-- let's draw a side view.

So here's our side view, and the car is moving with a velocity into the plane of the figure.

Now this surface here is frictionless.

And what we'd like to do is find out what speed the car can move such that it doesn't slide up or down the inclined plane.

So how should we analyze that?

Well our approach will be to apply Newton's second laws.

Now what's very important to realize is this is circular motion.

And for circular motion we know that the car is accelerating towards the center of the circle.

Now from the side view, towards the center of the circle is in this direction.

So the car is accelerating radially inward.

And that will guide how we choose our coordinate system.

And so we can then write our free body force diagram.

So let's begin with the analysis.

So we don't need to see the overhead view anymore.

So I'll just remove that, and then we can start drawing.

This is what we can refer to as our acceleration diagram.

And now let's draw the force diagram on the car as our choice of system.

So here's our angle, phi.

Because the acceleration was inward we're going to choose a radially outward coordinate and a vertical coordinate, K hat up.

Notice that this is different than just a mass on a fixed incline plane where we used unit vectors up and down the inclined plane.

The reason we choose our unit vectors like that-- to emphasize it again, is we already know this is constrain motion.

It's circular motion.

Now what is the free body-- what are the forces on the car?

Well there is the normal force, the plane on the car, and the gravitational force.

Now here-- whenever you're doing problems like this remember that the trig is crucial to get these angles right.

So that's phi, and that's phi, and that's our free body force diagrams.

And now we can write down Newton's second law.

So we'll start out with our usual approach, and we have two directions that we have to consider.

So in the radial direction there is an inward component of the normal force, like that.

And that's opposite the angle, so it's pointing opposite our direction.

So we have minus n sine phi.

The gravitational force is only in the negative K hat direction.

And we know that the acceleration is inward, and so there's a minus sign.

We have the mass, and the constraint for circular motion is that that's phi squared over r.

Where r was the radius of that circle, this can be thought of as the central point.

Now for the k hat direction, we have a component of the normal force that's pointing up.

I'll just draw that.

That's adjacent to the angle, so we have plus and cosine phi.

And we have the gravitational force downward, minus mg.

And as far as the vertical direction goes, because the car is going in a circle, there is no acceleration up or down in the vertical direction.

Again, that's a constraint in this problem.

That's equal to 0.

So in this problem, this is the side that we know, and we're trying to figure out up to the speed, v. Now, how do we analyze this problem?

Well you can see that if I write my two equations, this n sine phi equals mv squared over r.

And cosine phi equals mg.

We have two equations.

We have two unknowns, v and n.

Many times people just solve for n and try to find the equation-- and then substitute in, but you're also allowed to divide two equations, and that's much easier.

The masses cancel and we get the relationship, that tan phi is v squared over rg.

And so we have our result that the speed that the car can travel on a frictionless inclined plane and maintain uniform circular motion is exactly the square root of rg tan phi.

And that's how we analyze the motion of this car on a banked turn.

What we would now like to think about is what would happen if you're traveling faster or slower than this speed.

So suppose we have the prime bigger than the speed.

Now, what that means is that the car is going faster and the new equilibrium-- if you asked what would the radius be such that traveling at v primed the car undergoes circular motion, the prime would be equal to r prime g tan phi.

And so in order to go with this speed you have to go at a greater radius.

Now what does that mean?

Well, that means that if the car is traveling at v, so it's in this circular motion, and now the driver increases the speed to v prime, the car will start to slide up the inclined plane-- remember, it's frictionless-- until it reaches a-- as it starts slide up the inclined plane it will get to this new radius, r prime, but because a car will have a little inertia it will overshoot that speed, that radius, and then it will start to come back down the inclined plane, and it will oscillate about that point.

It won't be sinusoidal oscillations, but they'll be a periodic oscillation about this new radius, r prime.

The same thing, too, if we have the double prime less than d, then the double prime is equal to r double prime G tan phi.

Now remember, this double prime is not two derivatives.

I'm just using that as a notation to indicate different speeds.

So if the car is going along at speed, v, and slows down, what would happen is the new equilibrium radius is smaller so the car slides down the inclined plane until it gets to r double prime.

It turns out that it will overshoot that a little bit, and then start to move up.

And, again, it will oscillate around this new equilibrium length.

So on a frictionless inclined plane if you go faster then this speed the car slides up.

If you go slower than this speed, the car slides down.