## 13.6 Work-Kinetic Energy Theorem

There is a direct connection between the work done on a point-like object and the change in kinetic energy the point-like object undergoes. If the work done on the object is non-zero, this implies that an unbalanced force has acted on the object, and the object will have undergone acceleration. For an object undergoing one-dimensional motion the left hand side of Equation (13.3.16) is the work done on the object by the component of the sum of the forces in the direction of displacement,

$$W = \int_{x=x_i}^{x=x_f} F_x \, dx = \frac{1}{2} m v_f^{\,2} - \frac{1}{2} m v_i^{\,2} = K_f - K_i = \Delta K \qquad (13.6.1)$$

When the work done on an object is positive, the object will increase its speed, and negative work done on an object causes a decrease in speed. When the work done is zero, the object will maintain a constant speed. In fact, the work-energy relationship is quite precise; the work done by the applied force on an object is identically equal to the change in kinetic energy of the object.

**Example 13.7 Gravity and the Work-Energy Theorem**

Suppose a ball of mass $m = 0.2\ \text{kg}$ starts from rest at a height $y_0 = 15\ \text{m}$ above the surface of the earth and falls down to a height $y_f = 5.0\ \text{m}$ above the surface of the earth. What is the change in the kinetic energy? Find the final velocity using the work-energy theorem.

**Solution:** As only one force acts on the ball, the change in kinetic energy is the work done by gravity,

$$\begin{aligned} W^g &= -mg(y_f - y_0) \\ &= (-2.0 \times 10^{-1}\ \text{kg})(9.8\ \text{m} \cdot \text{s}^{-2})(5\ \text{m} - 15\ \text{m}) = 2.0 \times 10^{1}\ \text{J}. \end{aligned} \qquad (13.6.2)$$

The ball started from rest, $v_{y,0} = 0$. So the change in kinetic energy is

$$\Delta K = \frac{1}{2} m v_{y,f}^{\,2} - \frac{1}{2} m v_{y,0}^{\,2} = \frac{1}{2} m v_{y,f}^{\,2}. \qquad (13.6.3)$$

We can solve Equation (13.6.3) for the final velocity using Equation (13.6.2)

$$v_{y,f} = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2W^g}{m}} = \sqrt{\frac{2(2.0 \times 10^{1}\ \text{J})}{0.2\ \text{kg}}} = 1.4 \times 10^{1}\ \text{m} \cdot \text{s}^{-1}. \qquad (13.6.4)$$

For the falling ball in a constant gravitation field, the positive work of the gravitation force on the body corresponds to an increasing kinetic energy and speed. For a rising body in the same field, the kinetic energy and hence the speed decrease since the work done is negative.

**Example 13.7 Final Kinetic Energy of Moving Cup**

A person pushes a cup of mass 0.2 kg along a horizontal table with a force of magnitude 2.0 N at an angle of $30^{\circ}$ with respect to the horizontal for a distance of 0.5 m as in Example 13.4. The coefficient of friction between the table and the cup is $\mu_k = 0.1$. If the cup was initially at rest, what is the final kinetic energy of the cup after being pushed 0.5 m? What is the final speed of the cup?

**Solution:** The total work done on the cup is the sum of the work done by the pushing force and the work done by the friction force, as given in Equations (13.4.9) and (13.4.14),

$$\begin{aligned} W &= W^a + W^f = (F_x^a - \mu_k N)(x_f - x_i) \\ &= (1.7\ \text{N} - 9.6\times 10^{-2}\ \text{N})(0.5\ \text{m}) = 8.0\times 10^{-1}\ \text{J} \end{aligned} . \tag{13.6.5}$$

The initial velocity is zero so the change in kinetic energy is just

$$\Delta K = \frac{1}{2} m v_{y,f}{}^2 - \frac{1}{2} m v_{y,0}{}^2 = \frac{1}{2} m v_{y,f}{}^2 . \tag{13.6.6}$$

Thus the work-kinetic energy theorem, Eq.(13.6.1)), enables us to solve for the final kinetic energy,

$$K_f = \frac{1}{2} m v_f{}^2 = \Delta K = W = 8.0\times 10^{-1}\ \text{J} . \tag{13.6.7}$$

We can solve for the final speed,

$$v_{y,f} = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(8.0\times 10^{-1}\ \text{J})}{0.2\ \text{kg}}} = 2.9\ \text{m}\cdot\text{s}^{-1} . \tag{13.6.8}$$

## 13.7 Power Applied by a Constant Force

Suppose that an applied force $\vec{\mathbf{F}}^a$ acts on a body during a time interval $\Delta t$, and the displacement of the point of application of the force is in the $x$-direction by an amount $\Delta x$. The work done, $\Delta W^a$, during this interval is

$$\Delta W^a = F_x^a\,\Delta x . \tag{13.7.1}$$

where $F_x^a$ is the $x$-component of the applied force. (Equation (13.7.1) is the same as Equation (13.4.2).)