

Let's now consider our rolling wheel, and we want to look at some special conditions.

So at time t equals 0-- and we'll have our wheel that's rolling, here's the ground-- let's say that our point P is right up here at the top.

That's cm .

And we'll be in the ground frame now.

And then at a later time, time t , the wheel has moved to the right.

So let's draw the wheel over here.

Not the greatest picture of the wheel, but we'll have the wheel over here.

And now the point P has moved some angle $\Delta\theta$.

And we'll call this time interval Δt .

Now, the center of mass of the wheel has moved a distance.

X_{cm} is the velocity of the center of mass times Δt .

And the point P on the rim, this arc change, this length here on the rim that P has moved around in the center of mass frame, is $R \Delta\theta$.

Now, we want to ask ourselves-- we'll call this Δx .

We now have three possible conditions.

We call rolling without slipping.

That will be our first case 1.

And that's the case when the arc length Δs is exactly equal to the distance along the ground.

So we have ΔX_{cm} is Δs .

And so we get $V_{cm} \Delta t$ equals $R \Delta\theta$, or V_{cm} equals $R \Delta\theta$ over Δt .

Now, in the limit as Δt goes to 0, we have that $\Delta\theta$ over Δt in this limit as Δt goes to 0 is $d\theta/dt$. And that's what we called the angular speed.

So in our limit as this wheel is rolling without slipping, we have the condition that the velocity V_{cm} equals $R \omega$.

So that's our first condition, and we call this the rolling without slipping.

Now what is V_{cm} ?

V_{cm} ?

That's the velocity of the center of mass of the wheel, and every single point on this wheel has that same speed.

And $R \omega$, you can think of that as the tangential velocity in reference frame cm .

This is just the speed in the reference frame moving with the center of mass.

So this is our condition for rolling without slipping.

Now, our second case is imagine that the wheel is not moving forward at all, but it's just spinning.

That's what we call the wheel is slipping on the ground, for instance, if there were ice.

And so what we call slipping is a little bit more general.

It's whenever the wheel is spinning, and the arc length is much greater than the horizontal distance that the wheel has moved.

So we have Δs representing the arc length that the point has moved in the center of mass frame is greater than how far the center of mass is moving.

And so, again, we have $R \Delta \theta$ is greater than $V_{cm} \Delta t$, or in the limit $R \omega$ is greater than V_{cm} .

You can say it's spinning faster than it's translating.

And finally, the skidding condition.

Skidding-- imagine that the wheel-- you're braking a wheel.

The wheel is not spinning at all, but it's just sliding along horizontally.

So the horizontal Δx center of mass is bigger than Δs_{cm} .

And so this is the case where ΔX_{cm} , how far it moved horizontally, is greater than the amount of arc length that the point moved.

And so in the same type of argument, when we put our conditions in we get that V_{cm} is greater than $R\omega$.

And again, what that corresponds to in the skidding case, imagine the limit where it's not rotating at all, this would be 0, and it's just skidding along the ground, V_{cm} .

So we have our three conditions.

We have the slipping condition, where it's spinning faster than it's translating.

We have the skidding condition, where it's translating faster than it's spinning.

And we have the rolling without slipping condition, in which the arc length is exactly equal to the distance, horizontal distance, along the ground.