We've already talked about kinetic energy for an object that's rotating and translating.

Let's take, again, a simple object like a wheel.

Center of mass is moving, and we want to consider this is a bunch of point-like particles, th Jth particle with mass MJ, and a position vector RCMJ.

And now what we'd like to do is, what is the angular momentum of this about some point P?

And we're going to do the same type of decomposition that we've been doing right along.

Let's draw the vector RJ.

This is the vector from the point P to the object.

Here's the vector capital R, and there's the vector RCMJ.

And our same vector addition, R plus RCMJ.

j Now let's calculate the angular momentum about the point P. Now just let's recall two facts about the center of mass reference frame that we've already used.

The first is that the sum of the velocities, the momentum in the center of mass reference frame is zero, and if you integrate this equation in just exactly the same way, if you added up the mass times each of the position vectors, that's MJ, MJ, in the center of mass frame, that's also zero.

So we've used this when we talked about kinetic energy.

Now we're going to consider both of these results.

So recall that angular momentum about a point is the sum of the vector from the point, cross-product MJ, times the velocity BJ.

Now we can also use our law of addition velocities, VJ is V plus the VCMJ, and so now we're going to have to do both substitutions.

That's why this calculation is a little bit more complicated.

We have R plus RCMJ, cross MJ times capital D. That is the velocity of the center of mass with respect to the ground frame plus VCMJ.

Now we have four terms in the cross-product, and this time we'll write them all out.

So the first term is the sum over J of R cross MJ capital D.

The second term is, we'll take this one, and we'll cross with that one.

But because MJ is just a scalar, I'm going to pull it in front.

So that's MJ, RCMJ, and V is the same for every single particle in the object.

Remember, V is just the velocity of one reference frame with respect to the other.

So I can pull that out, cross V. And you're already noticing that this term will be zero.

The next term is when I take R cross MJVJ, V center of mass, J, summed over J, that's JR cross MJ, VCMJ.

But again, remember that R is the same vector.

It's just from P to the center of mass, so I didn't need to put the sum here.

I can pull R out of the sum and I have my sum like that.

And again, you're probably noticing that this term is zero and our final term is RCMJ cross MJ VCMJ.

So our final term is sum R, RCMJ cross MJ VCMJ.

Now from these two conditions, one and two, both of these two terms are zero.

And that's the power of using the center of mass reference frame.

So LP, now let's look at the first term.

Here, R is the same for every particle, V is the same for every particle, when we're summing over the mass, we're just getting the total mass, so the first piece is R cross M total V.

And this last piece is precisely the definition of the angular momentum of the Jth particle in the center of mass frame.

So that's the sum over J of LCMJ, because RCM cross MVJ R cross, say whatever speed this is in the center of mass frame, it could be that sum VCMJ.

BJ minus capital V.

This is exactly the angular momentum of that Jth mass in the center of mass frame, and when we total all of this up, we have R cross and total V plus LCM.

Now what this first term is, if you treated the whole object as a point mass, M total, moving with V, and here's our point P and there's R, this is just what we call the translational angular momentum.

It's treating the whole system as a point-like mass, and just calculate moving with speed V, and that's just the angular momentum.

We've called this V.

And this piece is the angular momentum about center of mass.

So sometimes, we can think of the angular momentum as corresponding to a translational, it has to do with the actual orbit of the object.

You could call this orbital angular momentum.

And this is the fact that the object is rotating.

You could think of this as the spin angular momentum about the center of mass.

This is the same type of decomposition that we saw with kinetic energy.

There was a translational component of kinetic energy and a rotational component of kinetic energy.

This is the analogous case for angular momentum about a point P.