

Today we'd like to explore the idea of the center of mass, or the center of gravity, of a rigid object.

For instance, take this rod.

And if I try to balance it on my finger at a certain point in that rod, I'm balancing it via the gravitational force.

And this point is often referred to as the center of gravity of the rod.

Now, if we were in empty space with no gravitational field, then center of gravity doesn't make any sense.

But this point still coincides with what we call the center of mass of the object.

And now I'd like to define center of mass.

So let's consider our rigid body.

And we'll just describe it as some object-- we'll make it idealized-- and that there's going to be trying to find some point in this object.

And we'll identify that point as the center of mass.

Now, let's imagine that this rigid body is made up of a bunch of little pieces.

So we have  $m_j$ .

And this  $j$  piece is located from the center of mass, a vector  $\mathbf{r}_{cm}$ .

Now how we want to define this particular point is that when we make the sum from  $j$  equals 1 to  $n$  over every single point in this body, then that will be 0.

And this will be the definition of the center of mass, that when you add up the position vector with respect to this point weighted by the mass, and you add up all of those vectors, you'll get 0.

Now, if you don't know where the center of mass is then this is difficult to calculate.

So let's find a way where you choose an arbitrary point, and that's write that arbitrary point, say, over here.

We'll write it like this--  $\mathbf{s}$ .

And we'll treat that as our origin.

And I'll draw a vector,  $\mathbf{r}_{sj}$ .

And here, I'll draw the vector  $R_{cm}$ .

And now what we have from our vector relationship is that the vector  $r_{sj}$  equals the vector  $R_{cm}$ -- and that's what I want to find-- plus the vector  $r_{cmj}$ .

And now let's add up-- multiply each of these by the mass and make a sum.

So we have  $m_j r_{sj}$  equals the sum of  $m_j r_{cmj}$ .

Now, the vector  $R_{cm}$ , this vector, no matter where we picked a point in this object, the vector's always the same.

And that's why I pulled it out of the sum.

And over here, I have the sum from  $j$  goes from 1 to  $n$ ,  $j$  1 to  $n$ , of  $m_j r_{cmj}$ .

Now, recall, this is precisely how we define the center of mass point, that this is 0.

And so we can now conclude that the center of mass-- so you pick an arbitrary point, and if you want to find that vector to the center of mass, what you do is you make the sum from  $j$  goes from 1 to  $n$  of  $m_j r_{sj}$ , and you divide that by  $j$  goes from 1 to  $n$ ,  $m_j$ .

And this is what we call the center of mass.

So conceptually, the center of mass is the point in the object where, if you take a vector to any of the little mass elements, and you weight it by the mass element, and you add them up, you get 0.

If you wanted to calculate the center of mass about any point, you choose a point,  $s$ , you draw the vector from  $s$  to the object.

You sum up those vectors.

We see by our vector triangle rule that we can now calculate that center of mass vector by this equation.

And now let's just rewrite this because this is the total mass.

And what we see is that the  $R_{cm}$  equals  $j$  goes from 1 to  $n$ ,  $m_j r_{sj}$  divided by the total mass.