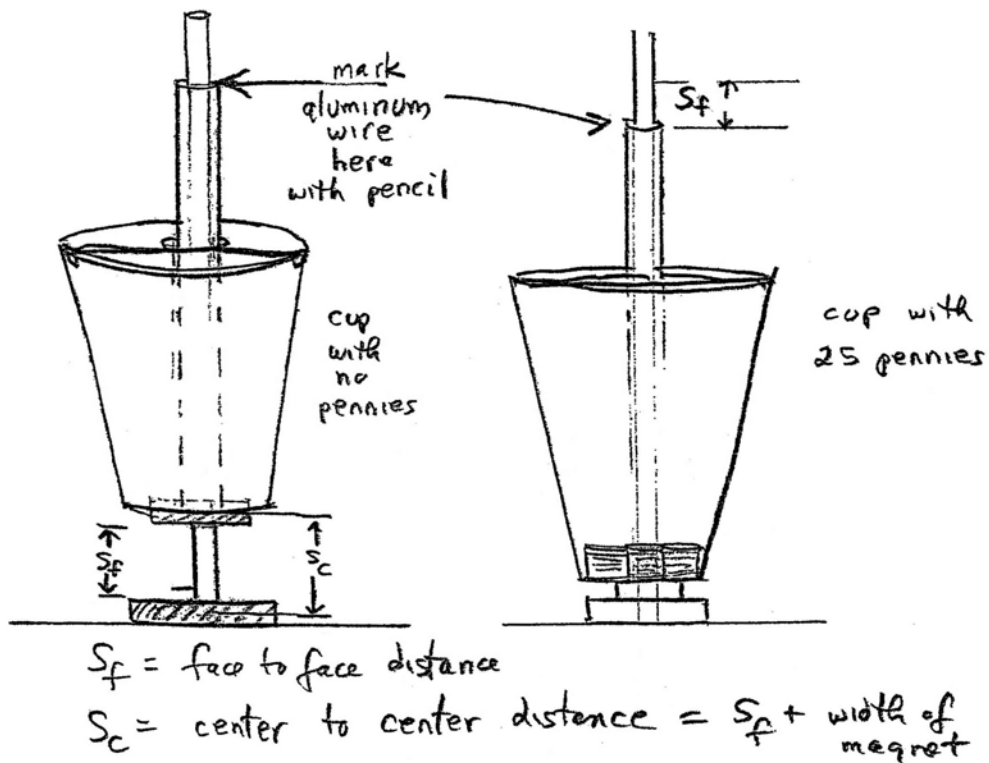


times. Give the cup a twist of say  $20^\circ$  —it should oscillate a few times, 3 or 4. Now remove the cup.

## Experiment

- The first measurement you will make are the width of the magnets. You should find this width to be about 4 mm.
- The second measurement you will make is the distance between the magnets without the weight of the cup and pennies acting on the suspended magnet. To do this, place your ruler on the block and measure the distance to the center of the upper magnet. You may notice that the upper magnet is tilted at an angle so try to estimate the location of the center of the upper magnet. Subtract half the width of a magnet (about 2 mm) in order to obtain the center distance  $S_c$ , between the faces. This distance corresponds to a force equal to the weight of 1 magnet,  $0.074 \pm 0.002$  N. Make this the first entry in your table of separation between magnet faces,  $S_c$  (in mm) and  $F$  (in N).



$$S_c = S_f + 4 \text{ mm}$$

- Place the upper magnet and cup on the wire. The magnets are now closer because of the weight of the cup, straw and centering wire,  $0.025 \pm 0.003$  N. Make a pencil mark on the aluminum wire next to the top of the straw.

- Put 5 pennies in the cup placed symmetrically around the straw at the bottom. Make another pencil mark on the aluminum wire next to the top of the straw.
- Continue adding pennies in groups of 5, always placing them symmetrically around the straw, and making pencil marks on the aluminum wire.
- As the magnets get close you may need to bend the aluminum wire slightly so that the magnet faces are parallel. With about 25 pennies the magnets are essentially touching and the center to center distance,  $S_c$ , corresponds to the width of a magnet (about 4 mm).

## Data

Remove the cup and upper magnet. Measure the distance between your highest pencil mark and the lowest pencil mark. This corresponds to the face-to-face distance between the magnets when there are no pennies in the cup. Add the width of one magnet to get the center-to-center distance,  $S_c$ . Continue this procedure after putting 5 pennies in the cup, measuring the distance between your highest pencil mark and your lowest pencil mark. This corresponds to the face-to-face distance between the magnets when there are 5 pennies in the cup. Add the width of one magnet to get the center-to-center distance,  $S_c$ . Estimate your measurements to the nearest 0.5 mm. Continue for 10, 15, 20, and 25 pennies.

Post 1982 pennies have a mass of 2.50 g and therefore a weight of 0.0245 N. Compute the cup and penny weights in newtons from their masses. Thus your cup, straw and centering wire mass, 2.50 g, should be added to the magnet mass,  $7.6 \pm 0.2$  g. For example, 5 pennies, with a mass 12.5 g yield a total mass of 22.6 g, corresponds to a force of 0.22 N.

Tabulate the data, with two columns labeled Force (N), and Center-to-Center Distance,  $S_c$ , (mm). Make two plots of the data; one on linear paper and the other on log-log paper with center-to-center distance  $S_c$  (in mm), along the horizontal axis and Force (in N) along the vertical axis.

## Analysis

On the log-log paper try to fit a straight line between the data points to match your best-fit curve. If you cannot match one straight line, you may be able to find two different regions where there are straight line fits. This means that the force between these magnets can be described by different inverse powers at different distances. In particular when they are very close and the separation is small compared to their width and length we often say they are infinite planes. Since any finite gap is in effect zero compared to infinity, you would not expect the force to vary—another way of thinking about this is

that with  $\infty$  featureless planes there is no scale of length and no way of telling how far away you are. This is why there is a slower variation of  $F$  with  $S_c$  when  $S_c$  is small. On the other hand, for very large distances one can expect  $F$  to go with some inverse power law.

Calculate the slope of the  $\log F$  vs.  $\log S_c$  best-fit straight lines. This gives the approximate power law for the force between the magnets for different ranges of center-to-center separation distance. You may use a program to find the best fit straight line. If you want to calculate the slope note that if the force is a power law

$$F = a(S_c)^b,$$

where  $a$  is a constant and  $b$  is the power. Then

$$\log F = \log(a(S_c)^b) = \log(a) + \log((S_c)^b) = \log(a) + b \log(S_c).$$

The slope of the  $\log F$  vs.  $\log S_c$  graph is the power exponent  $b$  and the intercept is the constant  $a$ . On the log-log graph paper choose two points that lie on your best-fit straight line. For example, suppose you choose the points

$$(x_1, y_1) = (9.0 \text{ mm}, 0.9 \text{ N})$$

$$(x_2, y_2) = (4.0 \text{ mm}, 3.0 \text{ N}).$$

Then the slope is

$$\text{slope} = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1} = \frac{\log(3.0 \text{ N}) - \log(0.9 \text{ N})}{\log(4.0 \text{ mm}) - \log(9.0 \text{ mm})} = \frac{\log(3.0 \text{ N} / 0.9 \text{ N})}{\log(4.0 \text{ mm} / 9.0 \text{ mm})} = -1.5$$

## Parts

- 1 wooden block
- 1 length #8 (1/8 in) aluminum wire, 140 mm
- 1 styrofoam cup
- 1 plastic soda straw
- 1 length #22 copper wire, 140 mm
- 1 paper clip #1
- 2 rectangular magnets with ~4 mm hole
- 50 US pennies

## Experiment CF Centripetal Force

Note: You will need both of your LVPS for this experiment. So build your second LVPS if you have not done so already.

### Introduction

This experiment is about centripetal force, the force that keeps objects moving in a circular path at a constant velocity. This force is always at right angles to the motion and directed towards the center of the circular orbit. By Newton's Second Law when a centripetal force acts on a mass, the force continually changes the direction of the velocity without changing its magnitude. This change in the direction of the velocity is always towards the center and accounts for the centripetal acceleration of the mass.

Examples of centripetal forces are tension in a string connected to a rotating mass; the gravitation force between a planet and a rotating satellite; and magnetic forces on moving charged particles in a magnetic field.

### Experiment

In this experiment, the centripetal force is the restoring force of a stretched rubber band with one end attached to the shaft of a dc permanent-magnet motor powered by your LVPS and the other end attached to a # 6-32 nut. The nut has a mass  $0.92 \pm 0.01\text{g}$  and the rubber band has a mass of  $0.22 \pm 0.02\text{g}$ . The motor whirls the nut in a circular path at a rotational frequency  $f$  that you will set using a stroboscopic method. You will measure the radius of the circular path,  $r$ , for various rotation speeds. This will allow you to calculate the centripetal acceleration

$$|a_r| = r 4\pi^2 f^2 .$$

You will set the rotational frequency by using a light-emitting diode (LED) that flashes 60 times per second when connected to the wall transformer. (The wall transformer supplies a nominal 12 Volts AC at 60 Hz). You can adjust the motor speed by adjusting the output voltage of the LVPS. You can then set your motor speed by a stroboscopic method. You will place a pattern of lines on the shaft of the motor. While the LED is directed downward onto the motor shaft, you adjust the LVPS until you see a stationary pattern of lines. This will correspond to a frequency that is some fraction of 60 Hz.

The rubber band stretches until the restoring force of the rubber band exerts the necessary centripetal force on the nut to keep the nut spinning in a circular orbit. You calibrate the rubber band by hanging pennies in a cup that stretch the rubber band and measure the stretched length of the rubber band as a function of the hanging weight. Thus by measuring the stretched length of the rotating rubber band (the radius of the circular orbit) you can use your calibration to

determine the centripetal force acting on the spinning nut. You can then verify that Newton's Second Law is indeed correct, namely that

$$|\vec{F}_{radial}| = m|\vec{a}_{radial}| = mr4\pi^2 f^2$$

## Theory

When an object is moving in circular motion the direction of velocity is always tangent to the circle. Therefore the direction of the velocity is always changing towards the center of the circle. This means that there is a non-zero component of the acceleration directed radially inward. This acceleration is called the *centripetal acceleration*. If our object is increasing its speed or slowing down there is also a non-zero *tangential acceleration*. But when the object is moving at a constant speed in a circle then only the centripetal acceleration is non-zero. Therefore there must be a '*centripetal force*', a radial force pointing inward, that produces this acceleration. Since Newton's Second Law is a vector equality,  $\vec{F} = m\vec{a}$ , can be applied to the radial direction yielding (in magnitude)

$$|\vec{F}_{radial}| = m|\vec{a}_{radial}|$$

## Kinematics of Circular Motion

We choose the origin of our coordinate system as the fixed central point. We shall choose coordinates for a point P in the plane as follows (Figure 1). One coordinate,  $r$ , measures the radial distance from the origin to the point P. The coordinate  $r$  ranges in value from  $0 \leq r < \infty$ . Our second coordinate  $\theta$  measures an angular distance along the circle. We need to choose some reference point to define the angle coordinate.

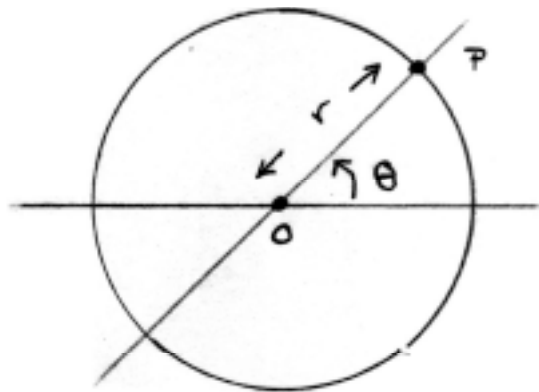


Figure 1: Polar Coordinate System

We choose a 'reference ray', a horizontal ray starting from the origin and extending to  $+\infty$  along the horizontal direction to the right. (In a typical Cartesian coordinate system, our reference ray is the positive x-direction). We define the angle coordinate for the point P as follows. We draw a ray from the origin to the point P. We define the angle  $\theta$  as the angle in the counterclockwise direction between our horizontal reference ray and the ray from the origin to the point P. All the other points that lie on a ray from the origin to infinity passing through P have the same value as  $\theta$ . For any arbitrary point, our angle coordinate  $\theta$  can take on values from  $0 \leq \theta < 2\pi$ .

### Velocity and Angular Velocity

The rate of change of angle with respect to time is called the angular velocity and is denoted by the Greek letter  $\omega$ ,

$$\omega \equiv \frac{d\theta}{dt}.$$

Angular velocity has units [rad/sec]. The component of the velocity  $v$  for circular motion is given by

$$v = r \frac{d\theta}{dt} = r\omega.$$

### Uniform Circular Motion

When the magnitude of the velocity (speed) remains constant, the object moves in *uniform circular motion*. Since the speed  $v = r\omega$  is constant, the amount of time that the mass takes to complete one circular orbit of radius  $r$  is a constant. This time interval is called the period and is denoted by  $T$ . In one period the object travels a distance  $s$  equal to the circumference,  $s = 2\pi r$ . Since the distance traveled in one period is the product of speed and period, we have

$$s = 2\pi r = vT.$$

Thus the period  $T$  is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega}.$$

The frequency  $f$  is defined to be the inverse of the period,

$$f = \frac{1}{T} = \frac{\omega}{2\pi},$$

The units of frequency are  $[\text{sec}^{-1}] = [\text{hertz}] = [\text{Hz}]$ .

### **Radial Acceleration**

The magnitude of the component of radial acceleration (*centripetal acceleration*) is given by

$$|a_r| = |v\omega|.$$

The direction of the radial acceleration is towards the center of the circle.

The centripetal acceleration can be expressed in several equivalent forms since the velocity and the angular velocity are related by  $v = r\omega$ . Thus we have for the magnitude of the centripetal acceleration two alternative forms:

$$|a_r| = r\omega^2 \quad (6.1.1)$$

and

$$a_r = \frac{v^2}{r} \quad (6.1.2)$$

Recall that the angular velocity is related to the frequency by  $\omega = 2\pi f$ . So we have a third expression for the magnitude of the centripetal acceleration

$$a_r = r4\pi^2 f^2 \quad (6.1.3)$$

A fourth form commonly encountered uses the fact that the frequency is defined to be the inverse of the period  $f = 1/T$ . Thus we have the fourth expression for the centripetal acceleration

$$a_r = r4\pi^2 f^2 \quad (6.1.4)$$

### **Experiment: Building the Apparatus**

Note: Do not plug in the wall transformer until you have completed the apparatus.

#### **Clamping the motor**

Take the 2-inch corner brace in the experiment kit and use the hose clamp to attach the motor to it as shown. Handle the motor carefully to avoid damage to the terminals of the motor. The clamp

should be tight enough to keep the motor from wiggling around, but not so tight as to keep it from turning freely. Use the 'C-clamp' to attach the assembly rigidly to your desk.

### Attaching the nut

Attach the 6-32 nut to the rubber band and the rubber band to the motor shaft by looping the rubber band over itself as shown. Loop it in the direction such that the rotation of the shaft tends to tighten the loop. Try to keep the two strands the same length and not twisted up.

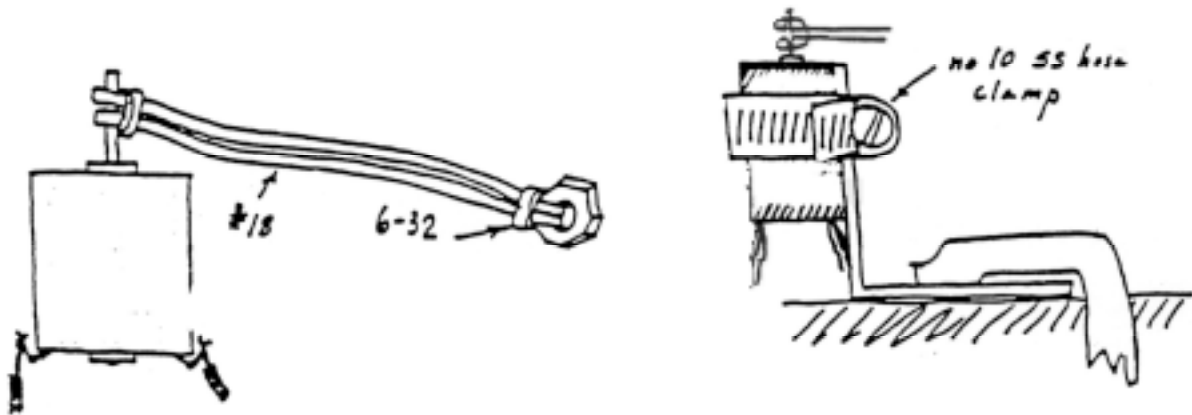


Figure 2: Clamping the motor and attaching the nut

### Attaching the stroboscopic pattern

Cut out the four-line pattern (you can find it on the last page of this write-up), make a hole in the center with a pushpin and press it onto the motor shaft above the rubber band.

### Connecting the LED

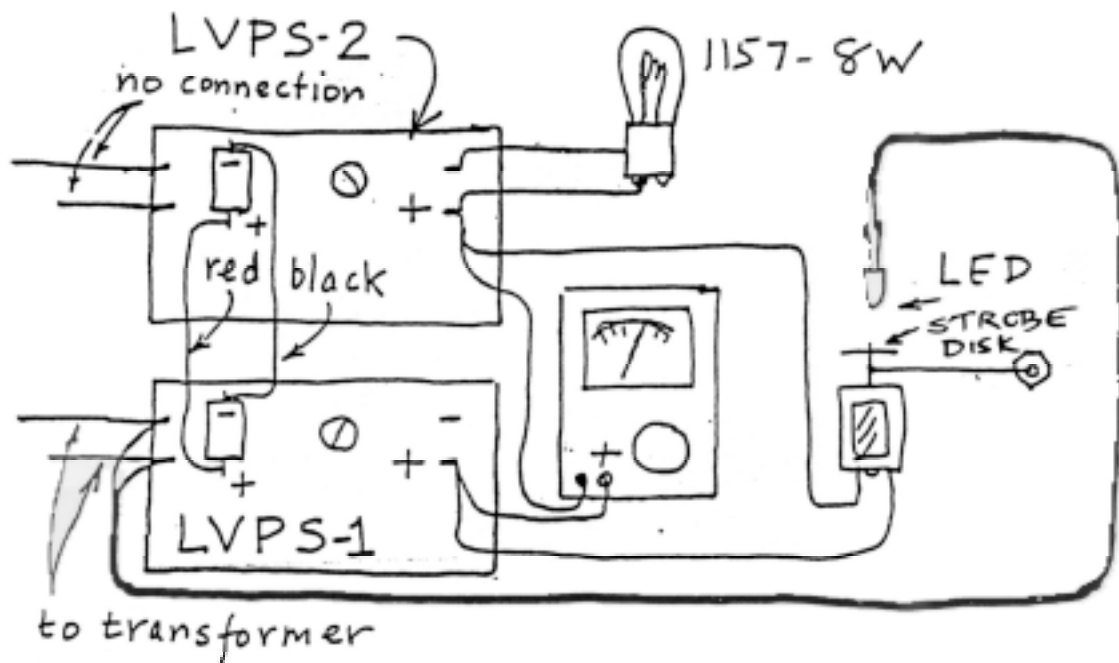
The LED needs an alternating voltage in order to flash so you will connect it to the ac input terminals of the rectifier in one of your partnerships' LVPS (call this one LVPS-1). Split the 'speaker wire' (two conductor stranded wire in transparent plastic insulation) an inch or so at each end of the 4 foot length provided in the experiment kit. Remove about 1/4 inch insulation from each of the ends and tin the exposed wire. Solder the pair at one end to the LED without twisting the wires. Use 1 1/2 inches of black tape folded lengthwise to protect the LED leads from shorting each other. Solder the pair at the other end to the ac input terminals of the rectifier in the LVPS-1. Again don't twist the wires.

### Connecting the two LVPS



Your LVPS only goes down to 1.5 volts, which is still too high to obtain the low rotational speeds you will need for this experiment. You will use your other LVPS (call this one LVPS-2) with the voltage reversed to adjust the voltage down to zero voltage.

You only need one wall transformer to drive both LVPS. You will do this by connecting the output of LVPS-1 rectifier to the  $1000\mu F$  capacitor of LVPS-2. Use a red clip lead to connect the positive sides of the  $1000\mu F$  capacitors of LVPS-1 and LVPS-2 together. Use a black clip lead to connect the negative sides of the  $1000\mu F$  capacitors of LVPS-1 and LVPS-2 together.



**Figure 3: Connecting the two LVPS**

### Connecting the 1157 lamp

You will use the 8-watt filament of your 1157 lamp for a current regulating effect that makes settings less critical and more stable. As the current increases in the filament the resistance goes up. This increased resistance tends to limit the increase in the current. Solder the leads of the 8 W filament of the 1157 lamp to the output terminals of the LVPS-2.

### Connecting the motor

Connect the motor leads to the plus terminals of the two LVPS with clip leads. By connecting the motor to the two positive terminals you are effectively subtracting the differences between the two voltage settings to get a net voltage across the motor between 0 and 1.5 volts. Connect

the input leads of the multimeter (MMM) at the 5 volt DC setting to the two positive output terminals of the LVPS so that you can measure the voltage across the motor.

## Doing the Spinning Nut Experiment

You will first measure the radius of the circular orbit of the spinning nut for various rotational frequencies, 10 Hz (six stationary lines), 12 Hz (five stationary lines), 15 Hz (four stationary lines), and 20 Hz (three stationary lines). Then you will calibrate the rubber band by stretching the rubber band with pennies in a hanging cup.

### Procedure:

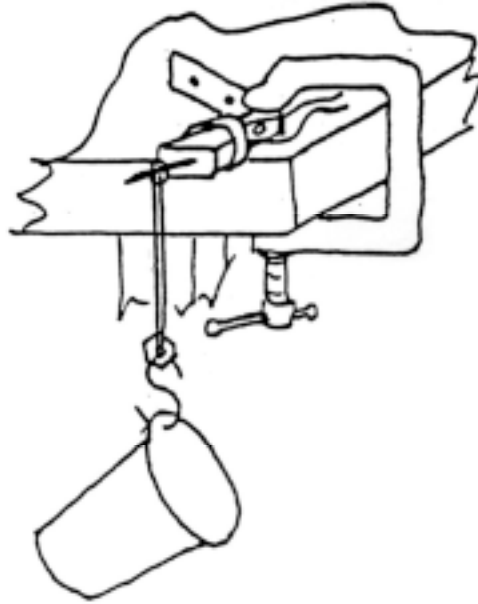
- **Unclip one motor lead and plug in the wall transformer.** Adjust the voltage of LVPS-2 to about 7.5 volts. This is enough to make the 8 W filament of the 1157 lamp glow brightly. Turn down LVPS -1 all the way and then reconnect the motor.
- Hold the nut so the rubber band is slightly stretched; the band should not rub on the motor housing. Turn up LVPS-1 so that there is 0.5 to 0.75 volts across the motor. You will see the shaft start to turn; immediately launch the nut into orbit. Start all over if the rubber band gets twisted up around the shaft.
- In order to see the stroboscopic pattern you will need to make the area immediately around the experiment dark. One partner can hold the LED about 1 inch above the motor and continually adjust the LVPS-1 until the 4-line pattern appears to be stationary. This means that (the simplest case) the band is advancing 90 degrees = 1/4 of a revolution each 1/60 of a second, and therefore the shaft is turning at a rotational frequency  $f = 15$  Hz.
- The other partner can measure the radius of the circular path of the spinning nut from the center of the shaft to the center of the nut.
- **Repeat these measurements for the various patterns corresponding to 10 Hz, 12 Hz, 15 Hz, and 20 Hz rotational frequencies.**

## Calibrating the Rubber Band

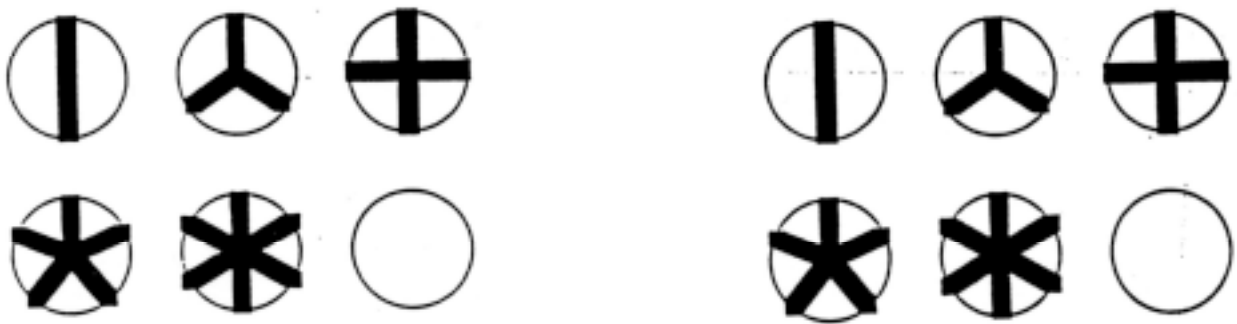
Clamp the corner brace on its edge with the motor shaft extended well out over the edge of your table or desk so that the rubber band and nut hang freely. Bend a paper clip into an S-shaped hook. Press one end through the paper cup just below its rim and hook the other end to the nut.

Now add pennies to the cup and measure the length,  $L$ , of the rubber band. Keep adding pennies to cover the range of lengths you measured when spinning the nut. The mass of the paper cup and the paper clip together is  $5.8. \pm 0.01g$ .

The weight required to stretch the rubber band to a length equal to the radius of the circular path of the nut is approximately equal to the centripetal force needed to accelerate the nut inward in its motion. We say "approximately", because the rubber band has mass and is stretched by the rotation even when there's no nut at the end. This isn't too serious, as you can see by spinning the rubber band by itself; it doesn't stretch much, and if it did, you could take data to make an appropriate correction. You should also note that excessive stretching of the rubber band could weaken it permanently; this could invalidate your results.



**Figure 4: Calibrating the rubber band**



**Figure 5: Stroboscopic line patterns**

## **PARTS LIST**

- 1 permanent magnet DC motor**
- 1 2" corner brace**
- 1 hose clamp #12 SS**
- 1 LED, wt. internal resistor**
- 4' #22 speaker wire**
- 2 nuts, 6-32 (5/16) steel**
- 2 rubber bands, #18**
- 1 paper clip, #1**
- 1 pushpin**
- 1 paper cup**
- 100 pennies**

## Experiment ET Energy Transformation & Specific Heat

We have introduced different types of energy, which help us describe many processes in the world around us: mechanical, electrical, chemical and heat energy. We have distinguished between energy stored in some system, potential energy: and energy associated with the motion of objects, kinetic energy. We have described some of the conversions or transformations in which one kind of energy turns into another kind, and we have pointed out that the final form is invariably heat. Consider Niagara Falls: the water's gravitational potential energy turns into kinetic energy as it falls. The turbines and alternators spin and electrical energy is generated. It's sent to you and your soldering iron turns it into heat, or your refrigerator turns it into mechanical energy to make the refrigerant flow to remove heat from the inside to the outside. There are an immense number of processes like that going on in both the man-made and the natural world.

In the Part I of the experiment you will calibrate an electrical temperature sensor, called a "thermistor" by immersing it in water whose temperature is measured with a glass thermometer. The thermistor (smaller and faster in response than the glass thermometer) is a compressed pellet of semi-conducting metal oxides whose resistance depends on temperature. Calibrating the thermistor means finding a relation between its electrical resistance and the temperature.

In the Part II of the experiment, electrical energy will be transformed into thermal energy resulting in the heating up of a sample of water. You will thus be able to find the specific heat,  $c_w$ , of water. The specific heat is the energy it takes to heat one kilogram one degree Kelvin (or equivalently one degree Celsius).

### Part I Calibrating the Thermistor

#### Apparatus

Start by soldering two wires, each about 1 foot long, to the thermistor.

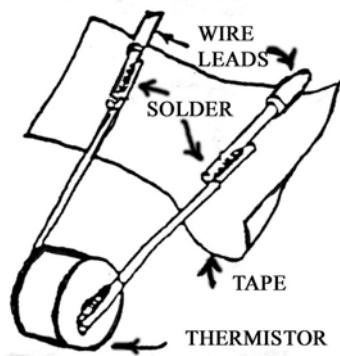


Figure 1: Thermistor

(It is a small cylindrical object with two leads; see Figure 1.) Use tape to prevent electrical shorts. Treat the thermistor gently, as the leads can be pulled or melted off rather easily.

Set the digital multimeter (DMM) to " $K\Omega$ " (kilo-ohms =  $1000\Omega$ ) and connect it to the thermistor with the clips or with your clip leads. A reading of about  $100\Omega$  should appear. Breathe on the thermistor and note its speed and sensitivity compared to the glass thermometer.

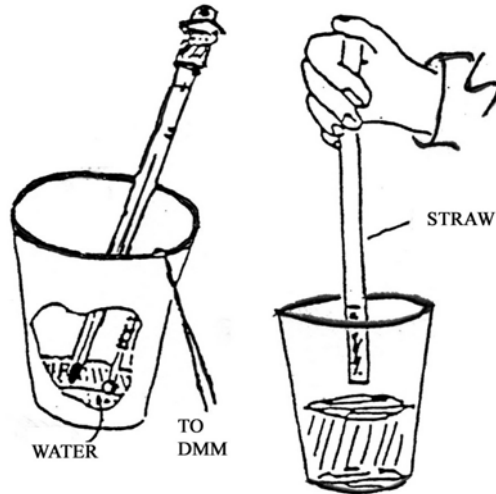
You will make many resistance readings with the DMM or the magnetic multimeter (MMM) and these use battery power. Get in the habit of turning off the DMM when not actually making resistance readings, and don't leave the MMM with its test leads connected for long periods.

Clear a work area so that if water spills there is no problem. Go to a nearby lavatory and fill two of your four styrofoam cups  $2/3$  full, one with hot water and the other with cold water. Do experiments in a third cup, and use the fourth to hold water when you're through with it. In these experiments you should watch out for heat sources like lights or heaters, and for drafts from windows and doors. It generally improves your measurements if you stir the water by gentle shaking to make sure it's well mixed and therefore at one temperature.

You can put the 1157 lamp into the hot water cup as an immersion heater to warm up water as needed. Connect the 8 W filament directly to the wall transformer (no LVPS). The 8W filament will maintain a cup of water near  $45^{\circ}\text{C}$ . We provide a lid for the experiment cup, which may be helpful in reducing heat loss, especially, that due to evaporation when using hot water.

### **Calibrating the Thermistor**

Put enough hot water into the third cup (the experiment cup, which serves as a calorimeter) to cover the end of the thermometer and the thermistor. Take and record temperature and resistance readings and also the time as the water continues to cool. Stir the water with the thermometer (or gently shake the cup) before each reading.



**Figure 2: calibrating the Thermistor**

When cooling slows, continue the calibration by adding cold water so as to cover the range from hot (45 C to 50 C) to cool (20 C to 25 C), taking data at about 10 intermediate values. You can use a soda straw as a pipette to add small amounts of water. **Plot the thermistor resistance versus temperature, with temperature along the horizontal axis. (See Data Analysis at end of lab write-up.)**

## **Part II Measuring the Specific Heat of Water**

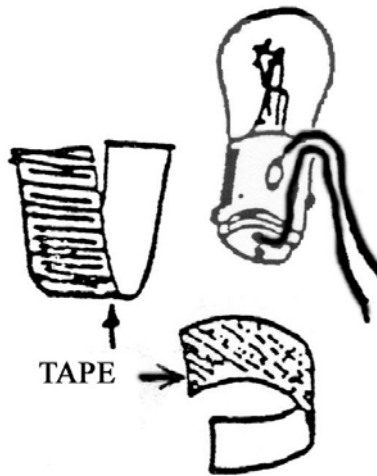
Now that you have calibrated the thermistor you can do the second experiment. You'll put the 1157 light lamp in a cup containing a known amount of water and measure the electric power delivered to the lamp and the rate of temperature rise of the water. This rate of rise, together with knowledge of the rate of energy input, will give you the specific heat,  $c_w$ , of the water in units of  $[J/kg \cdot K]$  if you assume energy conservation during the transformation. The actual value is  $c_w = 4185 J/kg \cdot K$ .

### **Apparatus**

The calorimeter is the experiment cup used previously. Fill it to a depth of 35 mm. You can mark this depth with a pen mark on the inside of the cup. Use a lid for the cup to prevent evaporation. Use your calibrated thermistor and the DMM to measure water temperature.

Before starting the experiment, stir the water with a glass thermometer, and take a temperature reading to check the correctness of your thermistor reading at one point.

Carefully fold back the wires soldered to the 1157 lamp as shown and tape over the end of the lamp base with black tape so as to insulate it electrically from the slightly conducting water in the cup. This will prevent current from flowing, which could affect the thermistor reading.

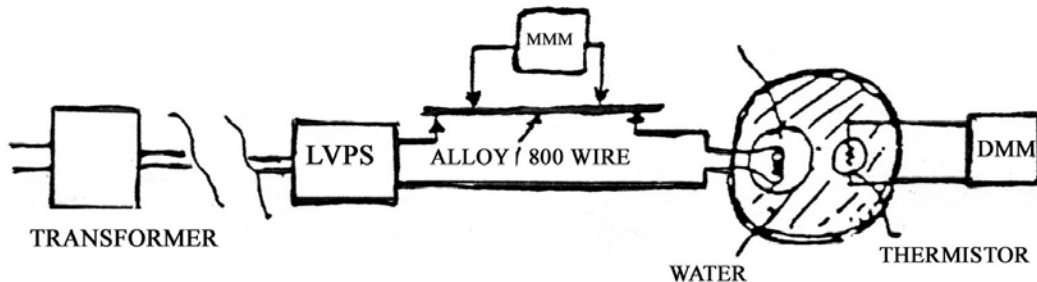


**Figure 3: Heat Source**

Locate the Alloy 800 resistance wire (stiff, thick, uninsulated wire of length  $\sim 0.3$  m). It's resistance is  $1.02\Omega$  per meter of length, so the resistance of your piece is about  $0.3\Omega$ . (Your DMM is quite inaccurate in this regime, but it will indicate  $0.3 - 0.5\Omega$ .)

### Measuring the Electric Power

Connect the 8 W filament of the 1157 lamp in series with the Alloy 800 resistance wire and the LVPS output (see sketch). Then disconnect the minus lead of the LVPS and plug in the wall transformer.



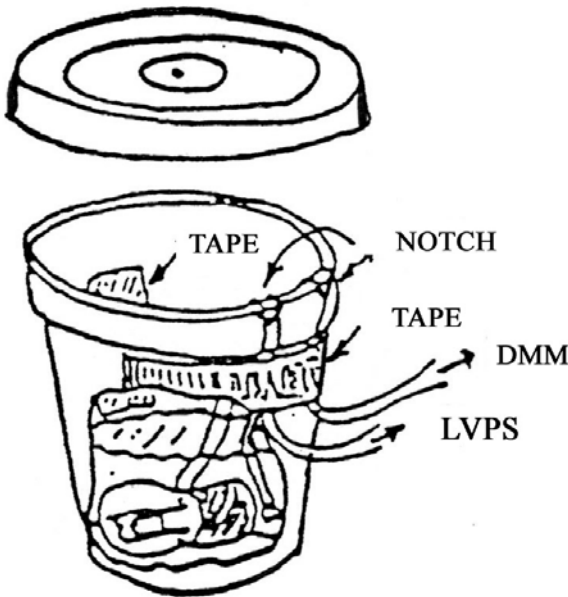
**Figure 4: Circuit Diagram for Experiment ET**

Use the MMM on the 25 VDC range to set the voltage of the LVPS to 10.0 V. Reconnect the minus lead of the LVPS and the lamp should light up but the regulator should keep the voltage at 10.0 V. Once the voltage is set, disconnect the MMM and rotate the MMM rotary knob to the " $50\ \mu\text{A}$  ( $250\ \text{mV}$ )" position on the DCA (DC Amperes) range, *not* the 250 mA position. The meter will then read 250 mV full scale; use the "250" scale on the face. Clip the MMM leads across a *carefully measured* length of the resistance wire (see sketch above), e.g. 0.25 m and **measure the voltage across this portion of the resistance wire**. Then unplug the wall transformer.



## Doing the Experiment

Put water (approx. room temp.) in the cup to a depth of 35 mm. Wedge the 1157 lamp (which tends to float) in the bottom of the cup. Make a small notch in the rim of the cup for the leads and pass them through the notch bending them and taping them in place (see sketch). Put the thermistor approximately in the middle of the water on one side of the lamp, bend its leads over the rim (again with a small notch), and tape. Make sure that the thermistor doesn't touch the cup or the lamp. Allow time for the entire apparatus to come close to room temperature. The thermistor readings give the water temperature. The thermometer can be used to keep track of room temperature. (As usual, watch out for drafts and unwanted sources of light and heat).



**Figure 5: The Calorimeter**

Press the lid onto the cup. You may need to make a few vertical cuts (say 4) around the cup rim to make it fit the lid. Work the lid on gently; a piece of tape will hold it on.

Disconnect the LVPS minus output lead. (You don't want to start heating the water until you're ready to take data.) Plug in the wall transformer, and set the LVPS output again to 10.0 V. Connect the DMM (set on the 250 mV range) across the same carefully measured points on the resistance wire.

## Taking Data

Since the lid on the cup prevents you from stirring with a thermometer, you should agitate the water by moving the cup in a horizontal circle about 1 to 1.5 inches in

diameter about twice a second, reversing direction every few seconds. The electrical connections had better be firm and the wires must have enough slack.

With the transformer plugged in, but the LVPS minus lead still not connected, start agitating and read resistances every 15 seconds. **One partner should agitate the water and keep track of the time. The other partner reads thermistor resistance values and records them with the time. When the readings are steady, or changing only 0.1 or 0.2Ω, reconnect the LVPS minus lead and note the MMM reading. Continue taking data every 15 seconds for 5 minutes. After 5 minutes, disconnect the LVPS, and continue agitating and recording resistances so as to get a value for the initial rate of cooling.**

## Data Analysis

### Part I: Fitting the Calibration Data

The decreasing exponential can fit the plot of your thermistor data,

$$R(T) = R_0 e^{-\alpha T}$$

where  $R(T=0) = R_0$  and  $\alpha$  are constants and  $R(T)$  is the resistance at Celsius temperature  $T$ . Note that  $R$  goes to 0 as  $T$  goes to 0. You can linearize this expression by taking the natural log (ln) of both sides,

$$\ln R = -\alpha T + \ln R_0$$

This becomes a linear equation if the dependent variable is taken to be  $\ln R$ . Your data, if truly exponential, should follow a straight line on a plot of  $\ln R$  versus  $T$ .

Plot  $\ln R$  versus  $T$  and fit a straight line to the data points (by eye and ruler). This yields the constants  $\ln R_0$  (the intercept at  $T = 0$ ) and  $\alpha$  (negative of the slope). The temperature  $T$  can then be obtained from

$$T = (\ln R_0 - \ln R) / \alpha$$

for any measured value of  $R$ . Check it for a couple of the values of  $R$  you measured to see if you get the proper temperatures.

### Part II Determining the Specific Heat of Water

#### Determining the Electrical Power of the Lamp

The measured voltage  $\Delta V$  across the resistance wire allows you to deduce the current flowing in the circuit. Suppose there is 0.25 m of resistance wire between the clips on the

MMM leads, and the measured voltage across the wire is  $\Delta V = 100 \text{ mV}$  (still using the "50  $\mu\text{A}$  (250mV)" setting and the "250" scale on the meter face). Ohm's law then gives

$$I = \frac{\Delta V}{R} = \frac{0.10 \text{ V}}{0.25 \Omega} = 0.40 \text{ A}$$

The power dissipated in the lamp is  $P = I\Delta V_L$  where  $\Delta V_L$  is the voltage across the lamp, i.e.,

$$P = I\Delta V_L = (0.40 \text{ A})(10.0 \text{ V} - 0.10 \text{ V}) = 3.96 \text{ W}$$

### Determining the Mass of the Water

You can estimate the mass of water in the calorimeter from its depth,  $d$ , before inserting the lamp and estimated 'average' radius (measured with one of your rulers). More precisely, use the bottom radius,  $r_1$ , and the top radius,  $r_2$ , of the cup and its height  $h$ . Here's the formula for the volume  $V$  of the water (can you derive it ).

$$V = \pi r_1^2 d + \pi r_1 \left( \frac{r_2 - r_1}{h} \right) d^2 + \frac{1}{3} \pi \left( \frac{r_2 - r_1}{h} \right)^2 d^3$$

For the styrofoam cup we provide, with  $r_1 = 2.0 \text{ cm}$ ,  $r_2 = 3.4 \text{ cm}$ , and  $h = 8.2 \text{ cm}$ , this yields a volume in [ $\text{cm}^3$ ]:

$$V(\text{cm}^3) = 12.6d + 1.07d^2 + 0.03d^3$$

From the measured depth  $d$  in cm and you can find the water volume  $V$  in  $\text{cm}^3$  and the mass in grams (numerically equal, closely enough). Convert your value for the mass of your volume into kilograms. Is your answer consistent with your first estimate?

### Analyzing the Data

Use the thermistor calibration to convert resistances to degrees C. Plot the temperature versus time, using an expanded ordinate scale that includes only the range of temperatures encountered, i.e., do not start the ordinate at  $0^\circ \text{C}$ ; start it at  $\approx 20^\circ \text{C}$ . Fit a straight line by eye which goes through the data points, and determine the slope  $dT/dt$  in units of [deg/s] for both the heating and cooling parts of your graph.

The quantity of heat  $Q$  required to increase the temperature  $\Delta T$  of a mass  $m_w$  of water is equal to

$$Q = m_w c_w \Delta T$$

where  $c_w$  is the specific heat capacity of water.

The rate that heat is flowing in time, the power, is the time derivative

$$P = \frac{dQ}{dt} = m_w c_w \frac{dT}{dt}$$

Treat the water as a system. Electrical power flows into the water via the 8W filament of the 1157 lamp.

$$P_{in} = I \Delta V_L$$

Power also flows out of the water during the heating process via the radiant heat loss, which you can approximate by

$$P_{out} \cong m_w c_w \left| (dT / dt)_{cooling} \right|.$$

This is approximate because the radiant power loss varies as a function of temperature and we are only using the power loss after the water has already been heated.

The power absorbed by the water is

$$P_{absorbed} = m_w c_w (dT / dt)_{heating} .$$

The difference between the power in and the power out is the power absorbed by the water,

$$P_{in} - P_{out} = P_{absorbed}$$

thus

$$I \Delta V_L - m_w c_w (dT / dt)_{cooling} \cong m_w c_w (dT / dt)_{heating} .$$

From your data you can obtain a value for the specific heat,  $c_w$ , of water in [J/kg · K].

## Error Analysis

Estimate the error on the slope you obtain as well as other input quantities to obtain an error for  $c_w$ . You may find that your experiment could have been done with more care regarding heat loss, agitating, etc. If so, try it again, it doesn't take long.

How serious was heat loss during your runs? (You might wish to devise a way to correct for it.) What is the approximate effect of the heat capacity of the light lamp, which has a mass of about 9 g (note that  $c_{glass} \approx 1000 \text{ J/kg} \cdot \text{K}$  and  $c_{brass} \approx 300 \text{ J/kg} \cdot \text{K}$ )? The effect of heat absorbed by the cup mass, 1.6 g is negligible.

## Experiment ET: Parts List

4	Styrofoam cups		1 ft	Resistance wire, alloy 800 (in Red box)
1	lid for same	1	#1157 lamp (in LVPST)	
1	drinking straw	1	thermometer (in Red box)	
1	thermistor		4	paper towels

# Experiment VS Vibrating Systems

## Introduction

Many vibrating or oscillating systems around us are visible or audible, but most are hidden. The motion of a clock pendulum, or of a child on a swing, can be seen. The vibrations of a bell, or of any musical instrument, are not easily seen but are certainly audible. The oscillations of current in the tuned circuits of a radio, or in atoms emitting light, are not directly perceived but can be measured or inferred.

These systems and others have several things in common: some quantities (position, sound pressure, electric field, etc.) vary sinusoidally in time at one or more natural frequencies depending on the pattern of motion or mode of vibration. In each cycle of vibration there is an exchange of energy from kinetic to potential and back, or from electrical to magnetic and back. In the case of mechanical oscillators the potential energy is associated with the work done against a linear restoring force provided by the deformation of the spring in a mass/spring system. In the case of the pendulum the potential energy is associated with the work done against the component of the gravitational force along the direction of the circular arc of a pendulum's bob. The pendulum is only an approximate oscillator for small angles of arc. The kinetic energy is associated with the moving mass.

In this experiment you'll investigate the properties of four mechanical oscillating systems, simple pendulum with two strings, simple pendulum with one string, mass/spring oscillator, and cantilevered beam. All these systems have nearly linear restoring forces.

## Mass/Spring Oscillator

In its ideal form, the mass/spring oscillator has a massless spring of spring constant  $k$ , attached to a point-like with mass  $m$  and a rigid support. We'll use rubber bands and the cup of pennies as an approximation. Clamp the piece of wood to a desk or table, and suspend from it a chain of rubber bands (use all that you have of one kind) with a paper clip at each end. Join the rubber bands as shown in Figure 1.

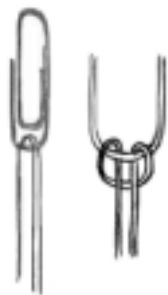


Figure 1: Rubber Band linkage

Hang the cup from the lower clip and measure the distance between the nearest points of the two paper clips (i.e., the length of the chain of rubber bands) for total suspended penny masses of about 10, 20, 50, 100, 120, 150, 180, 200, and 250 g.

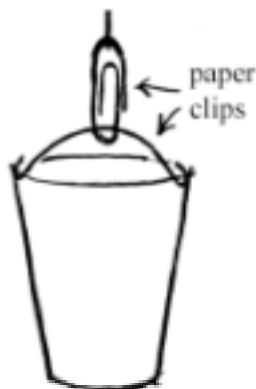
Note that the mass of a penny before and including 1981 is 3.1 g; from 1982 on it is 2.5 g. The paper cup has mass  $6.10 \pm 0.02$  g. Each paper clip has mass  $0.40 \pm 0.02$  g. Each rubber band has mass  $0.02 \pm 0.01$  g. Plot a graph of extension against suspended mass for this system. Is the graph linear?

Now measure the time for ten vertical oscillations and calculate the period of one vertical oscillation with a total suspended mass of about 150 g. Repeat for two different amplitudes of oscillation (one larger and one smaller).

Observe other possible oscillations -- pendulum and twisting motions -- of this oscillator. Look closely for any unusual features of the pendulum motion.

### **SIMPLE PENDULUM:**

We'll approximate the ideal "simple" pendulum -- a point mass on a massless inextensible string attached to a rigid support -- by a cup, containing pennies, attached by a long string (about 2 m) to a piece of wood clamped horizontally to a high support, for example a bookshelf, so that it projects well beyond the edge. Make a handle for the cup by first straightening out a paper clip. Pierce the ends of the paper clip through the top rim of the cup and then bend the ends of the clip with your needle nose pliers forming a jay shaped hook on each end. Attach the handle to the string using a paper clip (Figure 2).

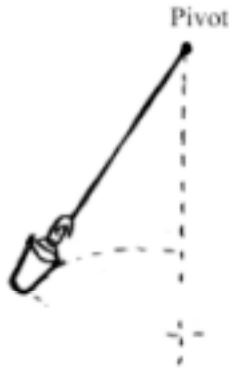


**Figure 2: Paper Cup Pendulum**

Suspend the cup, with 50 pennies, and time the period for swings of small amplitude -- say 10 degrees. Time a group of at least 10 swings. Don't forget to measure all lengths from the

attachment point to the center of gravity of the "bob" -- the pennies in the cup. Time how long it takes for the amplitude to decrease by 50%, and repeat with 5 pennies in the cup. Observe whether the period changes with the number of pennies in the cup. Repeat the period measurements with half the length of string.

Set the pendulum in circular motion forming a conical pendulum (Figure 3) and measure its period and other relevant parameters, for example the radius.



**Figure 3: Conical Pendulum**

### **EXPERIMENT VS -- PARTS LIST**

- 1 Length Twine
  - 1 Paper cup from Expt ET
  - 5 Rubber bands
  - 2 Paper clips
  - 1 Piece of wood  $1/2 \times 1/2 \times 10$
- Also C-clamp, and pennies from previous experiments.



# Experiment AM Angular Momentum

## Introduction

If an object, e.g., a heavy ball, comes straight at you and you catch it, you may stagger back while exerting the impulse -- some force for some time -- needed to bring its linear momentum down to zero. If you were on a frictionless surface you wouldn't be able to exert any forces to stop the ball, and you and it would move together, conserving linear momentum. Were you to catch the same ball as it passes beside you, with your arms outstretched to one side, you would have to exert an *angular* impulse, applying a *torque* for some interval of time, to bring its *angular* momentum down to zero. Suppose while catching the ball you were sitting on a stool firmly fixed to the ground but the seat of which could turn without friction. If you hung on to the ball, the ball and you would spin together indefinitely. If the stool were not anchored but free to slide and you caught the ball, then you, the ball and the stool would move in a combination of translational and rotational motion. Both the linear momentum and the angular momentum would be conserved during this collision since there would be no external forces or torques to provide impulses to change the total of each kind of momentum.

Just as conservation of linear momentum is a hidden part of many everyday happenings at all size scales, so is the conservation of angular momentum. The collisions of particles, the emission and absorption of radiation, the leaps and gyrations of cats, dancers, athletes, the motions of astronomical objects, all involve angular impulses and exchanges of angular momentum. In any isolated system angular momentum is conserved (as is linear momentum), despite mechanical energy losses ranging from zero (elastic collisions) to the maximum consistent with the conservation of linear momentum.

Part of the richness of phenomena and complexity of description in rotational dynamics comes from the fact that the same torque can be obtained with different combinations of lever arm  $\vec{r}$ , force  $\vec{F}$  and angle between them  $\theta$ . The same angular momentum can arise with different combinations of radius  $r$ , mass  $m$ , and velocity  $\vec{v} = \vec{\omega} \times \vec{r}$  or moment of inertia  $I$  and angular velocity  $\vec{\omega}$ . Some or all of these can change in any given situation.

## Experiments

You'll investigate two kinds of rotational phenomenon using a motor and a hub as a fixed axis of rotation.

In the first experiment you will measure the total frictional torque responsible for the slowing down of a rotating washer. You will place a washer on the hub. When the motor is turned on, the angular velocity of the shaft increases. When the motor is turned off, the total frictional torque decreases the angular velocity until the assembly comes to a stop. During the deceleration,

the motor will act as a generator providing a voltage that is a measure of the instantaneous angular velocity.

In the second experiment, you will measure the change in angular momentum due to an inelastic rotational collision in which a stationary washer is dropped on a spinning washer. During the collision there is a rotational frictional torque between the washers, slowing one washer down and speeding the other washer up until the washers are moving at the same angular velocity. The total angular momentum is nearly conserved during this collision. You will measure how closely angular momentum is conserved.

## Theory

When a **torque**,  $\vec{\tau}_S$ , is applied to a body about a point  $S$ , the body will acquire an *angular acceleration*,  $\vec{\alpha}$ . If the body is constrained to rotate about a *fixed axis of rotation* then the component of angular acceleration will be proportional to the component of torque about the axis,

$$\tau_S^{total} \propto \alpha,$$

where  $S$  denotes the point where the fixed axis passes through the center of the orbit. The constant of proportionality is called the *moment of inertia about the axis passing through  $S$* ,  $I_S$ , and it is a measure of how the mass is distributed about the axis of rotation,

$$\tau_S^{total} = I_S \alpha$$

The moment of inertia of a set of  $N$  masses about an axis of rotation is given by

$$I_S = \sum_{i=1}^{i=N} \Delta m_i (r_{\perp,i})^2$$

where  $r_{\perp,i}$  is the distance the  $i$ th mass  $m_i$  lies from the axis of rotation. For a continuous body the sum over all the masses becomes an integral over the body

$$I_S = \int_{body} dm (r_{\perp})^2$$

where  $r_{\perp}$  is the radius of the circular orbit of the mass element  $dm$ .

Let the angle  $\theta$  parameterize some point on the rigid body in a plane perpendicular to the axis of rotation. The *angular velocity*,  $\vec{\omega}$ , about the axis of rotation is the rate of change of the angle  $\theta$ , and the component is given by

$$\omega = \frac{d\theta}{dt}.$$

The component of angular acceleration is defined to be the rate of change in time of the component of angular velocity,

$$\alpha = \frac{d\omega}{dt}.$$

Consequently the applied torque will either increase or decrease the angular velocity.

The *angular momentum*,  $\vec{L}_S$ , of the mass about the axis of rotation passing through  $S$  is proportional to angular velocity,  $\vec{\omega}$ , with the moment of inertia  $I_S$  as the constant of proportionality,

$$\vec{L}_S = I_S \vec{\omega}.$$

Differentiating the above equation shows that the rate of change in time of angular momentum is equal to the applied torque

$$\frac{d\vec{L}_S}{dt} = I_S \frac{d\vec{\omega}}{dt} = \vec{\tau}_S.$$

When the total applied torque on a rigid body about some point  $S$  is zero then the angular momentum,  $\vec{L}_S$ , is conserved. If there is a constant applied torque,  $\vec{\tau}_S$ , over an interval of time  $\Delta t = t_f - t_0$ , then the change in angular momentum,  $\Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$ , which is known as the *angular impulse*, and is given by

$$\Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0} = \int_{t_0}^{t_f} \vec{\tau}_S dt.$$

The *rotational work*,  $W$ , which an applied torque,  $\vec{\tau}_S$ , does on a body in rotating that body about a fixed axis through an angle  $\Delta\theta = \theta_f - \theta_0$  is given by

$$W = \int_{\theta_0}^{\theta_f} \tau_S d\theta.$$

The rate of change in time of the work done by the torque is the *instantaneous rotational power* and is given by

$$P = \frac{dW}{dt} = \tau_s \omega.$$

## Apparatus

You have a permanent-magnet dc motor of the kind used in portable tape players which serves three purposes: (i) it provides a pivot, bearing or axis about which things can rotate with relatively low, but significant, friction, (ii) it is, of course, a motor which, when powered by your LVPS, can exert torque, rotate objects and give them angular momentum, and (iii) it can act as a DC generator that produces a dc voltage proportional to the angular velocity of its motor if it is made to rotate mechanically. In the latter mode, it can be used as an angular velocity sensor. The voltage generated by the motor is linearly proportional to the angular velocity of the motor,  $\omega = \beta V$ , where  $\beta$  is the constant of proportionality. You'll need a calibration of your generator to determine  $\beta$ , and we'll describe how to do this later. We'll use the motor in all these ways in this experiment.

You have a plastic bushing used to attach casters to tubular furniture, a wooden dowel in which we've drilled a hole, and a piece of rubber insulation. These can be assembled to make up a hub on which washers can be placed so that they can be rotated and so that their angular velocity can be measured.

The double pole, double throw slide switch (DPDT) will be used to connect the motor to the low voltage power supply (LVPS) when you want to accelerate a rotating object, and to the digital multimeter (DMM) when you want to measure angular velocity as the object coasts down.

To make readings at uniform intervals of, let's say, one second, you'll either have to call out seconds and record what your partner reads, or use the fact that the DMM flashes at a rate of two flashes a second.

## Assembling the Apparatus

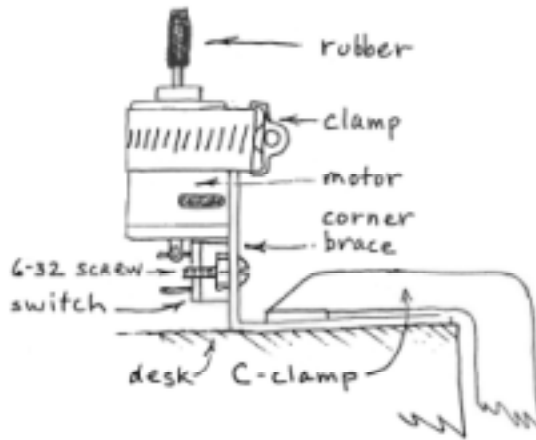


Figure 1: Overall assembly diagram

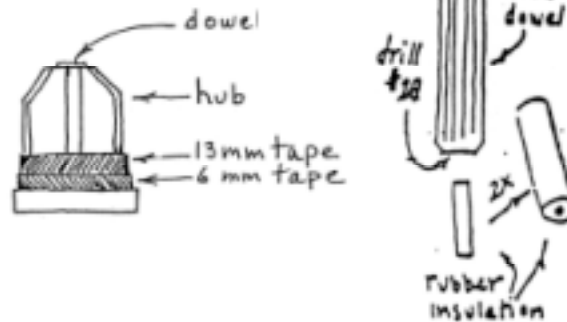


Figure 2: Hub diagrams

The assembly of the apparatus involves the following steps:

- construct the axis of rotation using the dowel, hub, tape, and rubber insulation as connector to shaft of motor (steps 1-4 below);
- connect the double pole double throw switch (DPDT) and low pass filter to motor (steps 5-7 below);
- connect the hub to shaft of motor clamped to table (steps 8-9 below);
- connect the LVPS to DPDT and motor (step 10 below).

1) Look at the white plastic object that will serve as a hub to carry the washers. It has a flat base and a conical top. Press the wooden dowel into the hub. The end of the dowel without a hole should be at the top of the hub and the end with the hole at the base.

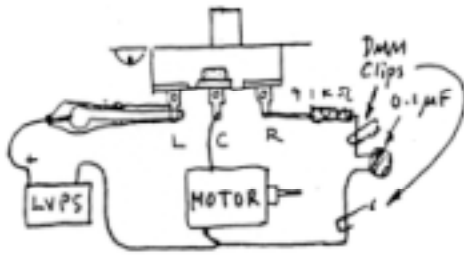
2) Cut off a length of black electrical tape about 300 mm long. Stick one end to your desk so that you can use scissors to cut the tape lengthwise into 2 strips, one narrow, about 6 mm wide, the other wider, 13 mm wide. Carefully wind the wider piece around the lower part of the hub, taking care not to cover the rim at the base. Now wind the narrow piece around the lowest part of the hub, again taking care not to cover the rim at the base. See if one of the washers can be pressed onto the taped hub. The washer should fit neither loosely nor so tightly that it's a struggle to press it on. Add or remove tape as appropriate to get the right fit.

3) Strip off a 12 mm piece of rubber insulation from the length of red 5 kV test lead in your kit. Look at the piece of insulation and remove any thread or strands of wire. What you want is

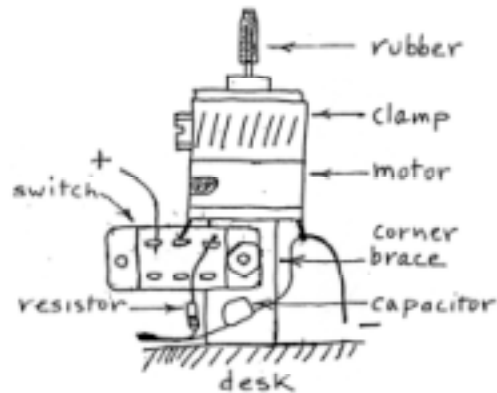
essentially a small piece of rubber tubing. Press it onto the motor shaft so that 9 mm are on the shaft and the rest isn't.

4) Clamp the motor to the corner brace with the stainless steel hose clamp. The clamp should be flush with the end of the corner brace and the top of the shaft end of the motor should be a few mm above that. Tighten the clamp firmly.

The double pole double throw switch (DPDT) will connect the motor alternately to the LVPS or to the DMM. The hookup diagram below shows the circuit.



**Figure 3: Hookup diagram for DPDT**



**Figure 4: DPDT Switch Connections**

- 5) Attach the DPDT switch to the corner brace below the motor using the 6-32 screw and nut.
- 6) One of the motor terminals may be so near the center terminal of the DPDT switch that you can simply solder them together. If not use a short length of bare wire to connect them.
- 7) Solder the 15 kΩ resistor to the end terminal of the DPDT switch that's right under the motor. Solder the 0.1μF capacitor from the free end of the resistor to other motor terminal. This low pass filter reduces commutator ripple that would otherwise cause wrong readings on the DMM. You will clip your digital multimeter (DMM) leads across the 0.1μF capacitor.
- 8) Press the hub and washer assembly onto the motor shaft with its rubber tube. It should be pressed on far enough so that no rubber is visible but still with some clearance (1 to 2 mm) so that nothing rubs on the brass bearing at the top of the motor.
- 9) Clamp the corner brace firmly to your desk or table with the C clamp.
- 10) Use clip leads to connect the negative output of the LVPS to the motor terminal that has one end of the 0.1μF capacitor soldered to it. Connect the positive output of the LVPS to the end terminal of the DPDT switch that has nothing soldered to it.

## Running the Experiment

The DPDT switch allows you to turn the motor on and off. When it's off the motor is connected to your voltmeter and acts as a generator whose output is proportional to its angular velocity.

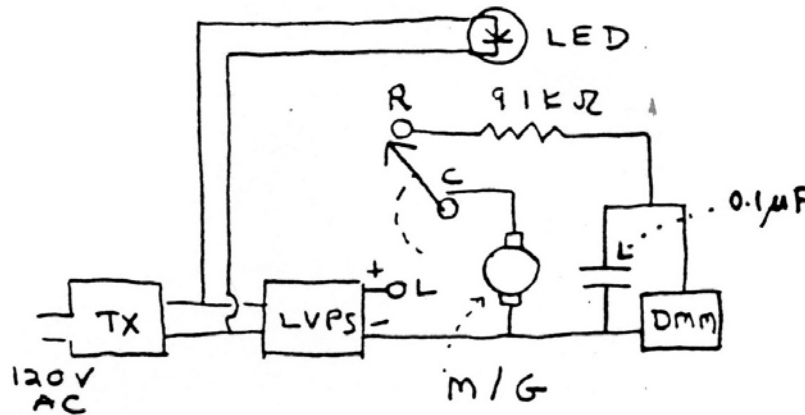


Figure 5: Circuit Diagram for Expt AM

Place one washer on the hub. (**Make sure you note the mass; it is printed in grams on the washer.**) Let the motor come up to speed, as you can tell by listening, operate the switch and observe the readings on your DMM. Since the voltage readings change twice a second on the DMM (sampling rate) they are hard to read. The first thing to do is blank off the end digit on the right with a piece of black tape so you have 2 or 3 digits to read instead of 3 or 4.

Make three runs in which you allow the motor to speed up and then coast down, taking voltmeter readings at regular intervals.

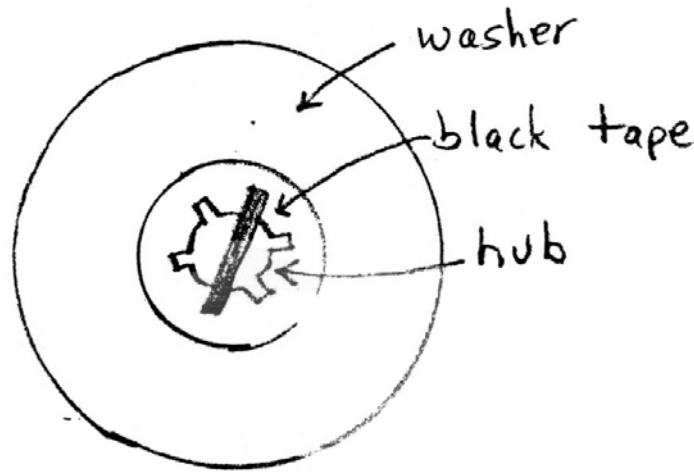
One person should read the meter and calls out numbers while the other writes them down (very rapidly) on a sheet of paper. This will not be easy; so practice and work out some procedure for taking reliable measurements at constant intervals of time.

## Calibration of the Motor/Generator

Connect the LED from Experiment CF to the 12V AC output from your wall transformer with leads sufficiently long to allow you to position the LED over your motor. (Note the transformer output will also be connected to the LVPS.)

Put a single washer on the hub. Place a small narrow piece of electrical tape completely across the top of the hub, as shown in figure 6. Since there are 60 flashes of light per second, the

tape should appear stationary when you reach a rotational speed of 30 Hz. (Wobble may make it sort of blurry.)



**Figure 6: Setting up the strobe pattern**

Accelerate the washer to as high a speed as you can achieve without the washer flying off the hub. Then disconnect the motor with the switch and read your voltmeter, as the motor coasts down. As the hub approaches 30 Hz, the LED will reflect off the tape at the top of the hub and the stroboscopic pattern will appear to slow down and spin the other way. (This will appear blurry but it is noticeable.) Just as the strobe pattern stops, you have reached 30 Hz. Record the value of the voltage. Repeat this three times and average your voltages. The angular velocity  $\omega$  of the motor is linearly proportional to the voltage  $V$  generated by the motor,  $\omega = \beta V$ , where  $\beta$  is the constant of proportionality. Recall that the angular frequency  $\omega = 2\pi f$ , where  $f$  is the frequency. Do not confuse the two quantities or their units:  $\omega$  [rad/sec] and  $f$  [Hz]. Calculate your value for  $\beta$ . From this data you can now calibrate your motor generator output so that you can determine angular velocity for any measured voltage with an assumption of linearity down to zero volts and zero Hz.

## Experiment One

With one washer on the hub, set the LVPS to 3.0 volts. Let the motor come up to speed. Turn the motor off and measure the voltages generated by the motor as it slows down (see above for ways to record the voltages).

Plot the voltage readings versus time for the three runs, arranging to have the first reading always at the zero of time. Because you are not starting each run at the same rotational speed the initial readings may differ, but one would expect the deceleration (represented by the slope) to be the same for each run.



## Experiment Two

Make three runs in which after about 6 s you drop a washer on the spinning hub, producing an inelastic rotational collision. Take data as before. Make sure you have at least three or four data points before and after the collision.

Plot voltage readings versus time (in arbitrary units) for the three runs on three sheets of linear graph paper. The collision takes about 0.5 s; that's the time during which the dropped washer is being accelerated and the one beneath decelerated by the friction torque between them. The meter reading process introduces additional uncertainty.

## Parts for Experiment AM

### From previous experiments:

C-clamp,  
2" corner brace,  
10-32 screws,  
2 LVPS,  
DC permanent magnet motor,  
stainless steel hose clamp,  
LED

### New Items:

1 5/16" hardwood dowel with #18 drilled hole  
1 1" plastic chair caster bushing  
2" 5-kV test lead wire  
2 1" US Standard Washer (2-1/2" OD, approx. 5/32" thick)  
1 paper clip #1  
1 slide type DPDT switch  
1 6-32 X 1/2 RH screw  
1 6-32 nut  
1 resistor 91 k $\Omega$  1/2 W  
1 capacitor, 0.1  $\mu$ F

# Experiment FL Flow

## Introduction

Steady flows are driven by forces that are balanced by resisting forces. For instance, the amount of water coming out of a shower depends on the water pressure as provided by private or municipal water systems, and the resistance of the many small holes in the shower head. Depending on the diameter of the holes and the velocity of the flow, the resistance can be due to viscosity (the friction of water against water when there are differences in velocity within the flow), momentum given to the fluid as it speeds up as it passes through the holes, and additional resistance when the flow becomes turbulent and there is vortex motion in the fluid.

## Experiment

You'll measure the rate at which water flows out of a container through a tube placed near the bottom. You'll do this for different length tubes. For each tube length you will calculate characteristic time constant for the flow rate. You will then compare these time constants as a function of the tube length. This experiment is primarily about taking data but later in the semester you will interpret your data to determine the viscosity of water.

## Apparatus

You have a 0.5 liter clear plastic bottle which you will cut to make a cylindrical container, four cups, one piece sugarless chewing gum, four stirrers (plastic tubes about 130 mm long and 2.8 mm inside diameter), two paper plates, one push pin, one 8 penny nail, one 1 1/2" length poly tube, and paper towels. You also have a photocopy of a ruler on your parts list. You can use thermometer in your Red Box that will be used in Expt ET. You'll need either a stopwatch, or a clock or watch with a sweep second hand for timing the drop in water height.

Use the attached three copies of *Data Sheet for EXPERIMENT FL: FLOW* for experiments I, II, and III, with 3, 2 and 1 stirrers, respectively. There are spaces for recording the length of the tube, the diameter of the tube (see parts list), room and water temperatures at start and finish of each experiment, and a table with six columns: one for water level (the "head"), three columns for three sets of time measurements, one for the calculated average time, and one for the standard deviation of the average time. Record any relevant circumstances, phenomena, troubles, etc. in the space below labeled *notes*.

There are attached to the write-up four pieces of linear graph paper and three pieces of semi-log graph paper although you may plot your data using a computer.

## Procedure

Open up the cap of the bottle and empty the contents. You should have one piece of sugarless chewing gum, four stirrers (plastic tubes about 130 mm long and 2.8 mm inside diameter), one and one half inch length of poly tube, one 8 penny nail, rubber bands, and a push pin.

You will first connect three stirrers together with small pieces of poly tubing. Cut a 15 mm piece of the poly tubing and place the end of one stirrer into the tubing (Figure 1).



**Figure 1: Stirrer inserted into poly tube**

Fit a second stirrer into the other end of the tubing until the stirrers meet together in the middle of the poly tube (Figure 2). This step is delicate, you can easily rip the stirrer.



**Figure 2: Two stirrers connected by a poly tube**

Now cut another 15 mm piece of the plastic tubing and connect a third stirrer to the other two. **Measure and write down the length of this tube. (It should be about 390 mm).**

Cut the plastic bottle with scissors about 15 mm from its bottom so as to make a cylindrical container that tapers down to the screw cap at the end. In order to cut the container begin by making a hole with the push pin and enlarging it with the nail so that you can get the scissors in to cut the bottle.

Make sure the cap is screwed on tight. Turn the container upside down and put it in one of the cups. You will make a hole in the container at a point that is just above the rim of the paper cup. Mark a spot for your hole with a pen. This hole will be about 65 mm from the bottom of the cap.

(You can now take the container out of the cup).

Start making the hole with a push pin. Enlarge this small hole with the point of the nail, turning the nail like a drill. The stirrer should just fit; press it in about 10 mm. (If you wish, you can practice beforehand making holes in the otherwise useless cut off part of the container). Use a small amount of chewing gum to seal around the plastic tube and black tape or gum to seal undesired holes. If necessary, cut away the rim of the cup to make room for the gum seal.

You'll need to measure the depth of water in the container in 10 mm steps to within  $\pm 1$  mm or less. You have a photocopy of a ruler on the side of your Experiment Flow parts list. Attach the ruler to the side of the container with rubber bands so that zero of the metric scale is at the center of the hole and so that it extends upward about 90 mm.



**Figure 3: Container, ruler, and stirrer inserted in hole**

Your apparatus will consist of the upside down container in a cup; an upside down second cup that supports the tube; and a third cup that will collect the outflow. Make sure that the stirrers are straight. Figure 4 shows the apparatus when two stirrers are connected together.



**Figure 4: Apparatus with two stirrers**

Remove items from your desk that might be damaged by water and arrange your apparatus as shown in figure 4 but with three stirrers instead of two. Sand, pebbles or coins might be put in the bottom of the container support cup to make it sit steadier. Use the paper plates under the container support cup and the collector cup to contain spills. Paper, folded or wadded, can be used to adjust heights so that the tube is reasonably level.

### **Taking Data**

Your first data measurements will use a tube made from three stirrers. You will fill the container and then record the time it takes for the water level in the container to pass successive 10 mm (1 cm) scale marks. You will repeat the experiment three times. Then you will detach the third stirrer and repeat the process making three more trial runs. You will then detach the second stirrer and repeat the process making three more trial runs.

Fill a cup with water as close to room temperature as possible, as indicated on your thermometer that you will find in your red box. **Record room and water temperatures at the start and finish of your experiment. Record the length of your stirrers. Record your results on the accompanying Data Sheet for Experiment Flow.**

Fill the container and observe what happens as the water level drops. You and your partner should decide on a method for taking data. For instance, one partner can call out as the water level passes successive 10 mm (1 cm) scale marks and the other writes down the time to the nearest second as the level passes each mark. Refill the container using the collector cup, keeping water from flowing until you're ready by putting a finger lightly over the end of the tube. Have another cup at hand to catch any extra flow. Note that at some level the nature of the flow

changes from a continuous stream to a series of drops. Either stop timing or, if you wish, record the level and time corresponding to this change. **Take data for three trial runs. Record your results on the accompanying Data Sheet for Experiment Flow.**

Repeat the above procedure after shortening the plastic tube from 3 stirrers (about 390 mm) to 2 stirrers (about 260 mm) and then to 1 stirrer (about 130 mm). **Take data for three trial runs for two stirrers and take data for three trial runs for one stirrer. Record your results on the accompanying Data Sheet for Experiment Flow.**

## **Averaging**

For each combination of stirrers, you have three time measurements for each height that the water level passes. Average the three times to give an average time corresponding to each water level. **Record your results on the accompanying Data Sheet for Experiment Flow.**

## **Graphing the Data**

You have four linear and three semi-log graph papers. In addition you have one linear graph paper for the time constants vs. length of tube graph.

On three linear graph papers, **plot the level of the water above the hole (the "head") in mm versus the average time in seconds to reach that level for each of the tube lengths.** (You should have three graphs.)

On the semi-log paper, **plot the head vs. average time for each of the tube lengths.** On semi-log paper, the horizontal axis is a normal linear scale, but the vertical axis is marked off in proportion to the logarithms or natural logarithms of the numbers represented. (Recall logarithms in base 10 are proportional to natural logarithms according to  $\log_{10} u = \ln u / \ln 10$ .) So you can choose the numbers 1,2,3 etc on the vertical axis to represent 10 mm, 20 mm, 30 mm, etc. So a data point like (30 mm, 55 sec) is placed at the 3 on the vertical axis and at the 55 on the horizontal axis. You should have three semi-log graphs. Draw the best straight lines through the points as judged by eye. (When choosing the 'best straight line', consider which points are most reliable. Are the first and last measurements as reliable as the others?)

## **Reporting the Data**

Your linear graph of head vs. average time should be an exponentially decaying function,

$$h(t) = h_0 e^{-\alpha t} \quad (10.1.1)$$

where  $h_0$  is the value of the head at  $t = 0$  and  $\alpha$  is a constant. The time constant  $\tau$  associated with this exponential decay is defined to be the time that it takes for the head to reach a value of  $h(\tau) = h_0 e^{-1} = h_0 / 0.368$ . Since  $h(\tau) = h_0 e^{-\alpha\tau}$ . The time constant  $\tau$  is related to the constant  $\alpha$  according to

$$\alpha\tau = 1 \text{ or } \tau = 1/\alpha .$$

Your semi-log plot should be nearly a straight line. The natural logarithm of Equation 10.1.1 is

$$\ln h(t) = \ln h_0 - \alpha t .$$

So a plot of  $\ln h(t)$  vs. time  $t$  will be a straight line with

$$\text{slope} = -\alpha = -1/\tau .$$

## Finding the Time Constant

### Method 1

Obtain the time constant  $\tau$  for the flows with the three tube lengths by the following procedure that will use results from both your semi-log graph or your linear graph. Use your best straight line in the semi-log paper graph to determine the value of the head,  $h_0$ , at  $t = 0$  for each of the three experiments. (Note: if you just choose your initial value from your data sheet you are ignoring the rest of your data values.) You can obtain the time constant from the linear graph of head vs. average time by directly reading off the time that the head reaches the value  $h_0 / 0.368$ . Determine the time constants for each of your three experiments. **Report your results in the table Time Constants for Experiment Flow that is attached to the write-up.**

### Method 2

From your semi-log plot, calculate the slope of your best-fit straight line. Compute the time constant according to  $\text{slope} = -\alpha = -1/\tau$ .

**Make a plot of tube length vs. time constant. Is there any nice curve that passes through the data points? What does the extrapolation to zero tube length mean?**

## Data Sheet for Experiment Flow

Expt. I: Length of tube =

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

Notes:



## Data Sheet for Experiment Flow

Expt. II: Length of tube =

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

Notes:

## Data Sheet for Experiment Flow

Expt. III: Length of tube =

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

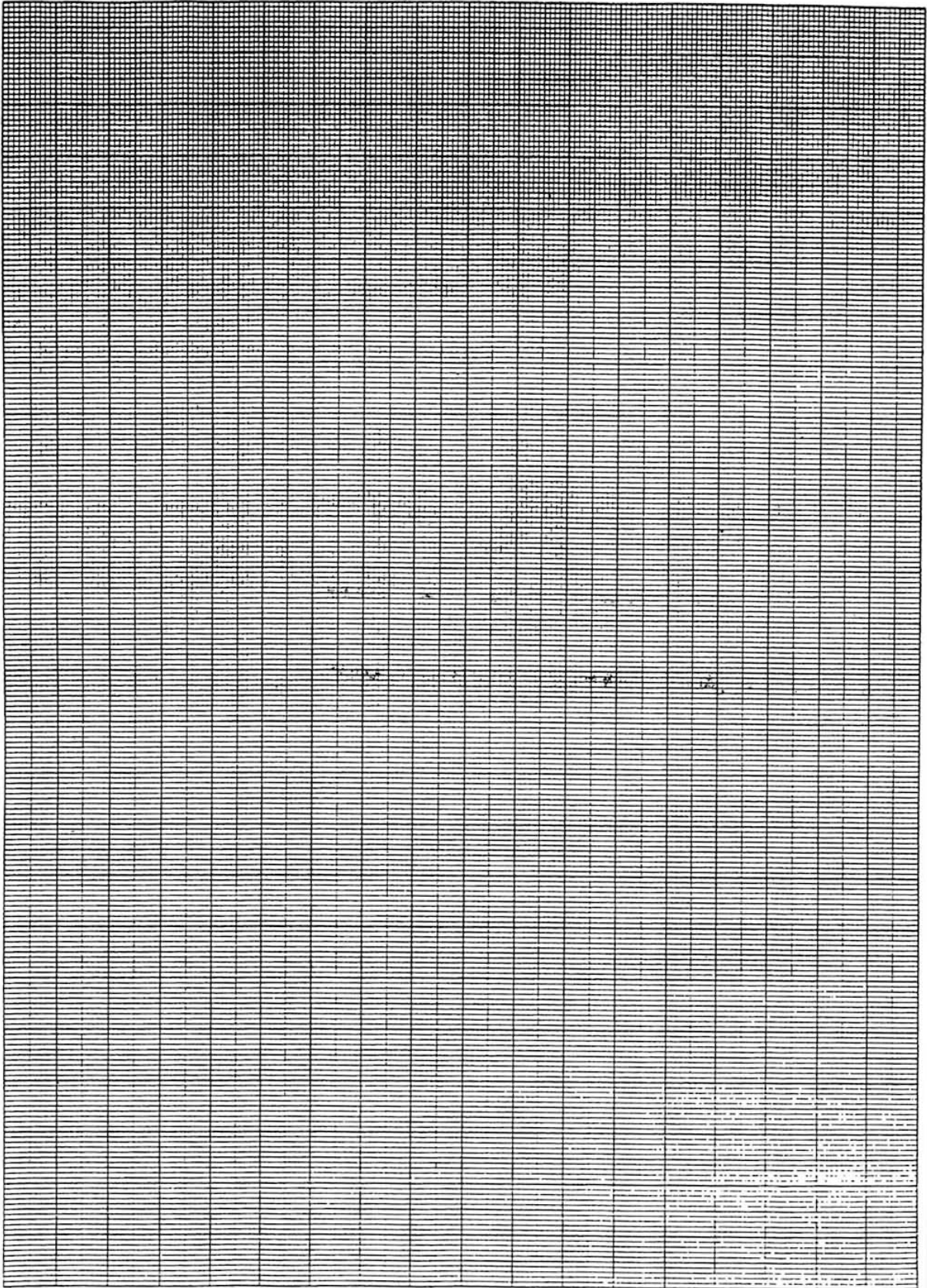
Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

Notes:

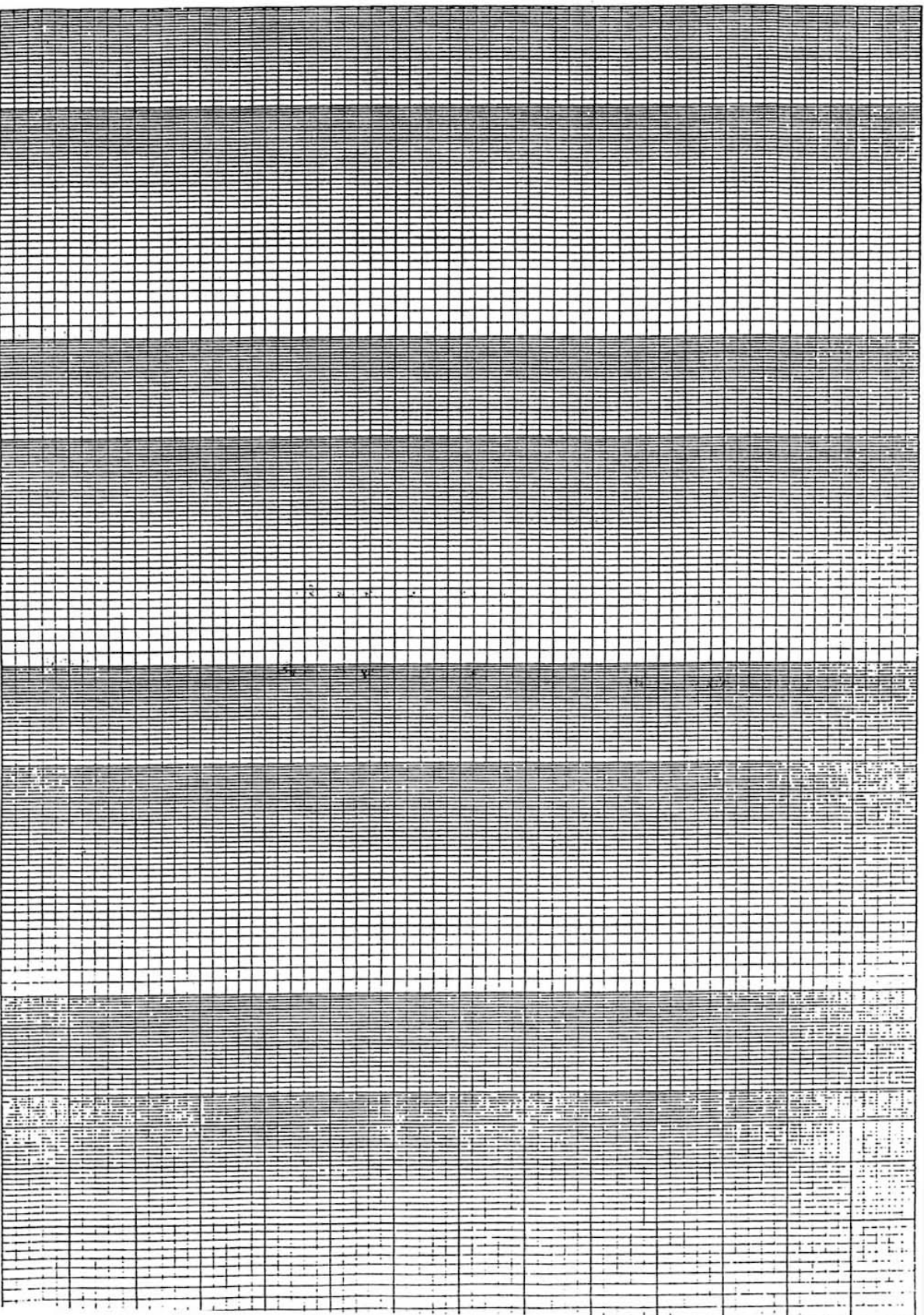
### Time Constants for Experiment Flow

Trial	Tube length (mm)	Time constant (sec)	Container diameter (mm)	Temperature start ( $^{\circ}\text{C}$ )	Temperature finish ( $^{\circ}\text{C}$ )
Experiment I					
Experiment II					
Experiment III					

Notes:



10  
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## **Red Box and Toolkit Contents**

### **Red Box Top Tray**

- 1 magnetic multimeter
- 1 digital multimeter
- 1 transformer
- 1 thermometer
- 1 roll electrical tape
- 1 meter/transformer kit
- 1 C-clamp
- 1 piece of resistance wire
- 1 clip lead kit
- 100 US pennies
- 1 piece of emery cloth
- 1 length, spare hookup wire

### **Bottom of Red Box**

- 2 LVPS

1 each: AM, CF, ES, ET, FM, FO, LVPS, LVPST, VS

wood block, 1/2" pipe, brass rod, paper cups, paper towels

### **Toolkit**

- 1 soldering iron
- 1 tube solder
- 1 pr. slip joint pliers
- 1 pr. long nose pliers
- 1 wire stripper
- 1 pr. scissors
- 1 small slotted screw driver
- 1 large slotted screw driver
- 1 phillips screw driver
- 1 12" plastic ruler
- 1 6" steel ruler
- 1 tape measure

## Kits Parts List (\* in Red Box)

### Clip lead kit

- 6 black insulated alligator clips
- 6 red insulated alligator clips
- 3' #22 black stranded wire
- 3' #22 red stranded wire

### Meter/Transformer kit

- 4 black alligator clips
- 2 red alligator clips
- 1 AA cell
- 3 spare fuses
- 2 spare button cells (old style meters only)

### ES

- 1 knife switch (DPDT)
- 1 AA cell
- 1 battery holder
- 3 wiring nuts
- 2 10 meg  $\Omega$  resistors
- 1 20  $\Omega$  resistor
- 1 1.0  $\mu$ F polyester capacitor

### LVPS

- 1 Perfboard
- 4 rubber feet
- 1 full wave bridge rectifier
- 1 1000  $\mu$ F electrolytic capacitor
- 1 socket for LM317 regulator
- 1 5 k $\Omega$  potentiometer
- 1 1 $\mu$ F, electrolytic capacitor
- 1 390  $\Omega$  1/2W resistor
- 1ft. #22 tinned copper wire
- 1 LM317 voltage regulator
- 1 heatsink for LM317 reg AM
- 2 1" plated steel washers
- 1 1" plastic caster bushing
- 1 hardwood dowel with #19 drilled hole
- 2" 5kV test lead wire
- 1 paper clip
- 1 slide type SPDT switch
- 1 6-32 X 1/2 screw
- 1 6-32 nut
- 1 91 K $\Omega$  1/2 W resistor

1 0.1  $\mu$ F ceramic disc capacitor

### **LVPST**

1 resistor 2.4  $\Omega$ , 2 W

1 #1157 tail lamp

### **FO**

1" 1/8" Al rod

1 wire nut

8" 1/16" brass rod\*

1' #22 stranded wire

2 plastic stirrers

2 paper clips

2" 5k test lead

1 paper cup\*

2 cable clamps

3 #6 washers

2 #6 sheet metal screws

1 pre-drilled wood block\*

1 5/8" binder clip

1 alligator clip

1 solder lug

1 6-32 machine screw

1 6-32 nut

12" 1/2" PVC pipe\*

1 10M $\Omega$  resistor 1/2W, 5%

1 1.0  $\mu$ F polyester cap.10%

### **FM**

2 magnets

1 paper cup\*

1 piece aluminum rod

1 soda straw

1' solid wire

1 pre-drilled wood block\*

### **CF**

1 12V DC motor

1 2" corner brace

1 hose clamp #12 SS

1 LED, wt. internal resistor

1 .1 $\mu$ F ceramic disc capacitor

4' #22 speaker wire

2 nuts, 6-32 (5/16) steel

2 rubber bands, #18

1 paper clip,#1



- 1 push pin
- 1 paper cup (in red box)

### **ET**

- 4 #6 cups
- 1 cup lid
- 1 soda straw
- 1 thermistor
- 4 paper towels (in red box)

### **VS**

- 1 paper cup ( in red box)
- 1 6' piece of string
- 1 10" hacksaw blade
- 1 fender washer
- 1 4-40 X 1/2" screw
- 1 4-40 nut
- 6 #18 rubber bands
- 1 12" piece of wood
- 2 thumbtacks
- 2 paper clips

### **AM**

- 2 1" plated steel washers
- 1 1" plastic caster bushing
- 1 hardwood dowel with #19 drilled hole
- 2" 5kV test lead wire
- 1 paper clip
- 1 slide type SPDT switch
- 1 6-32 X 1/2 screw
- 1 6-32 nut
- 1 91K $\Omega$  1/2W resistor
- 1 0.1 $\mu$ F ceramic disc capacitor

### **FL**

- 1 500ml plastic bottle
- 4 plastic stirrers
- 1 1 1/2" length poly tube
- 4 paper cups
- 2 paper plates
- 4 paper towels
- 1 push pin
- 1 piece of gum (sugarless)
- 1 2 1/2" nail
- 2 rubber bands