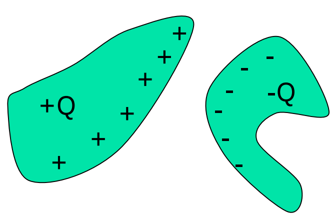
8.022 (E&M) – Lecture 6

Topics:

- More on capacitors
- Mini-review of electrostatics
  - (almost) all you need to know for Quiz 1

Last time…

- Capacitor:
  - System of charged conductors
- Capacitance: $C = \frac{Q}{V}$
  - It depends only on geometry



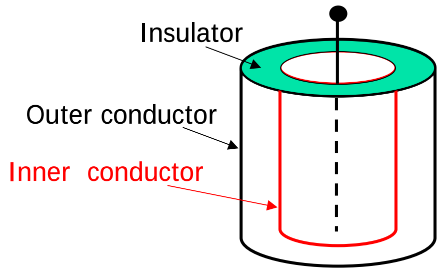
- Energy stored in capacitor:
  - In agreement with energy associated with electric field

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

- Let's now apply what we have learned…

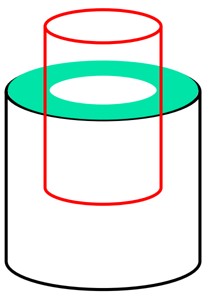
Wimshurst machine and Leyden Jars (E1)

- A Wimshurst machine is used to charge 2 Leyden Jars
- Leyden Jars are simple cylindrical capacitors



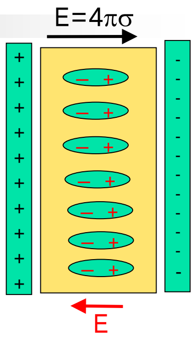
- What happens when we connect the outer and the outer surface?
- Why?

Dissectible Leyden Jar (E2)



- A Wimshurst machine is used to charge a Leyden Jar
- Where is the charge stored?
  - On the conductors?
  - On the dielectric?
- Take apart capacitor and short conductors
  - Nothing happens!
- Now reassemble it
  - Bang!
- Why?
  - Because it's "easier" for the charges to stay on dielectric when we take conductors apart or energy stored would have to change:
    $U=Q^2/2C$, and moving plates away C would decrease → U increase

Capacitors and dielectrics



- Parallel plates capacitor:

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{A}{4\pi d}$$

- Add a dielectric between the plates:
  - Dielectric's molecules are not spherically symmetric
  - Electric charges are not free to move

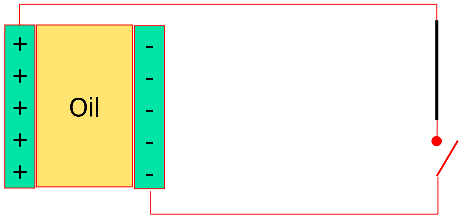
→ **E** will pull + and – charges apart and orient them // **E**

- $E_{dielectric}$ is opposite to $E_{capacitor}$
  - Given Q → V decreases
  - Given V → Q increases

} → C increased!

Energy is stored in capacitors (E6)

- A 100 μF oil filled capacitor is charged to 4KV
- What happens if we discharge it thought a <u>12" long iron wire</u>?



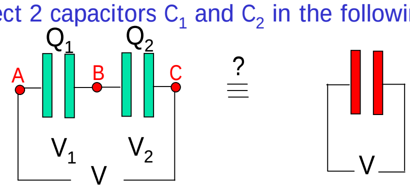
- How much energy is stored in the capacitor?
  - U= ½ CV^2 = 800 J Big!
- Resistance of iron wire: very small, but >> than the rest of the circuit

→ All the energy is dumped on the wire in a small time

→ Huge currents! → Huge temperatures! → The wire will explode!

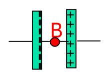
Capacitors in series

- Let's connect 2 capacitors C_1 and C_2 in the following way:



- What is the total capacitance C of the new system?

$$V_1 + V_2 = V$$
$$Q_1 = Q_2 = Q$$
$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1 + V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

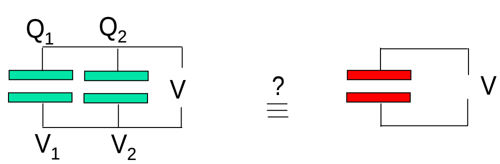


This is 1 conductor that starts electrically neutral
→ $Q_1=Q_2$

$$C = \left(\sum_i \frac{1}{C_i}\right)^{-1}$$

Capacitors in parallel

- Let's connect 2 capacitors C_1 and C_2 in the following way:



- What is the total capacitance C of the new system?

$$V_1 = V_2 = V$$
$$Q_1 + Q_2 = Q$$
$$C = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V_1} + \frac{Q_2}{V_2} = C_1 + C_2$$

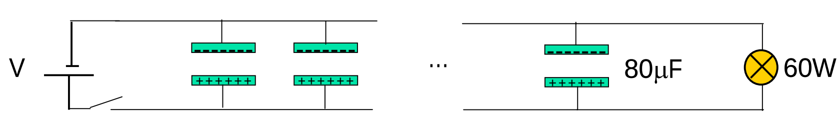
→ $$C = \sum_{i=1}^{i=N} C_i$$

Application

- Why are capacitors useful?
  - …among other things…
  - They can store large amount of energy and release it in very short time
- Energy stored: $U = \frac{1}{2} CV^2$
  - The larger the capacitance, the larger the energy stored at a given V
- How to increase the capacitance?
  - Modify geometry
    - For parallel plates capacitors $C = A/(4\pi d)$: increase A or decrease d
  - Add a dielectric in between the plates
  - Add capacitors in parallel

Bank of capacitors (E7)

- Bank of 12 x 80 μF capacitors is parallel



  - Total capacitance: 960 μF
- Discharged on a 60 W light bulb when capacitors are charged at:
  - V = 100 V, 200 V, V = 300 V
- What happens?
  - Energy stored in capacitor is $U = \frac{1}{2} CV^2$

    $\rightarrow V = V_0 : 2xV_0 : 3xV_0 \rightarrow U = U_0 : 4xU_0 : 9xU_0$
  - R is the same → time of discharge will not change with V
  - The power will increase by a factor 9! ($P=RI^2$ and $I=V/R$)
  - Will the bulb survive?
    - Remember: light bulb designed for 120 V…

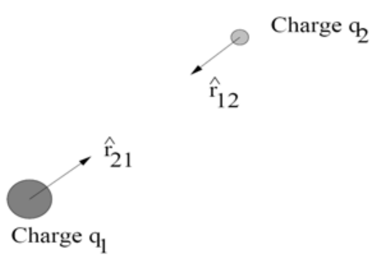
Review of Electrostatics for Quiz 1

Disclaimer:

- Can we review all of the electrostatics in less than 1 hour?
  - No, but we will try anyway…
- Only main concepts will be reviewed
  - Review main formulae and tricks to solve the various problems
  - No time for examples
    - Go back to recitations notes or Psets and solve problems again

The very basic:

Coulomb's law



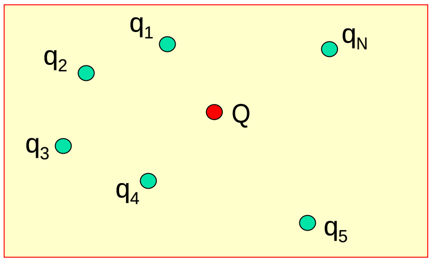
$$\vec{F}_2 = \frac{q_1 q_2}{|r_{21}|^2}\hat{r}_{21}$$

where F_2 is the force that the charge q_2 feels due to q_1

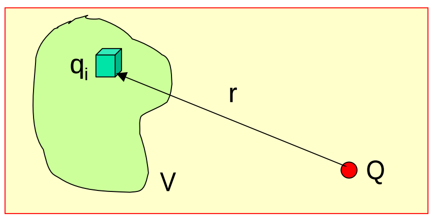
NB: this is in principle the only thing you have to remember: all the rest follows from this an the superposition principle

The very basic:

Superposition principle



$$\vec{F}_Q = \sum_{i=1}^{i=N} \frac{q_i Q}{|r_i|^2} \hat{r}_i$$

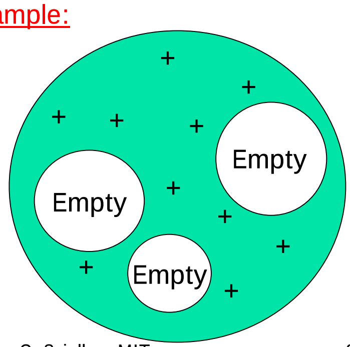


$$\vec{F}_Q = \int_V \frac{dq\ Q}{|r|^2} \hat{r} = \int_V \frac{\rho\ dV\ Q}{|r|^2} \hat{r}$$

The Importance of Superposition

Extremely important because it allows us to transform complicated problems into sum of small, simple problems that we know how to solve.

Example:



Calculate force F exerted by this distribution of charges on the test charge q

q

Electric Field and Electric Potential

- Solving problems in terms of $F_{coulomb}$ is not always convenient
  - F depends on probe charge q
- We get rid of this dependence introducing the Electric Field

$$\vec{E} = \frac{\vec{F}_q}{q} = \frac{Q}{|r|^2}\hat{r}$$

- Advantages and disadvantages of **E**
  - **E** describes the properties of space due to the presence of charge Q ☺
  - It's a vector → hard integrals when applying superposition… ☹
- Introduce Electric Potential ϕ
  - ϕ(P) is the work done to move a unit charge from infinity to P(x,y,z)

$$\phi(x, y, z) = -\int_{\infty}^{P} \vec{E} \bullet d\vec{s}$$

NB: true only when ϕ(inf)=0

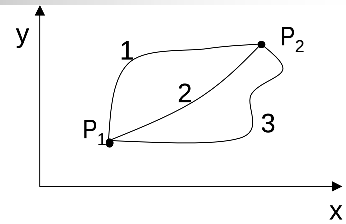
  - Advantages: superposition still holds but simpler calculation (scalar) ☺

Energy associated with **E**

- Moving charges in E requires work:

$$W_{1->2} = -\int_1^2 \vec{F}_C \bullet d\vec{s}$$

$$\text{where} \quad F_{Coulomb} = \frac{Qq\hat{r}}{r^2}$$



- NB: integral independent of path: force conservative!

- Assembling a system of charges costs energy. This is the energy stored in the electric field:

$$U = \frac{1}{2}\int_{\substack{\text{Volume}\\\text{with}\\\text{charges}}} \rho\phi dV = \int_{\substack{\text{Entire}\\\text{space}}} \frac{E^2}{8\pi} dV$$

Electrostatics problems

- In electrostatics there are 3 different ways of describing a problem:

$\rho(x,y,z)$

$E(x,y,z)$ $\phi(x,y,z)$

- Solving most problem consists in going from one formulation to another. All you need to know is: how?

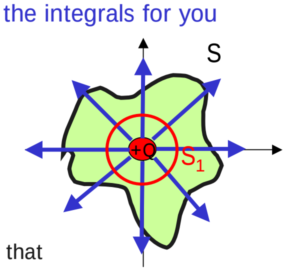
From ρ → E

- General case:
  - For a point charge: $\vec{E} = \frac{q}{|r|^2}\hat{r}$
  - Superposition principle: $\vec{E} = \int_V d\vec{E} = \int_V \frac{dq}{|r|^2}\hat{r}$

Solving this integral may not be easy…

- Special cases:
  - Look for symmetry and thank Mr. Gauss who solved the integrals for you
    - Gauss's Law: $\Phi_{\vec{E}} = 4\pi Q_{enc}$

$$\oint_S \vec{E} \bullet d\vec{A} = 4\pi \int_V \rho\, dV$$



    - N.B.:
      - Gauss's law is always true but not always useful: Symmetry is needed!
      - Main step: choose the "right" gaussian surface so that E is constant on the surface of integration

From ρ → φ

- General case:
  - For a point charge: $\phi = \frac{q}{r}$
  - Superposition principle: $\phi = \int_V \frac{dq}{r}$

NB: implicit hypothesis: φ(infinity)=0

The problem is simpler than for **E** (only scalars involved) but not trivial...

- Special cases:
  - If symmetry allows, use Gauss's law to extract **E** and then integrate **E** to get φ:

$$\phi_2 - \phi_1 = -\int_1^2 \vec{E} \bullet d\vec{s}$$

  - N.B.: The force is conservative → the result is the same for any path, but choosing a simple one makes your life much easier....

From φ to **E** and ρ

Easy! No integration needed!

- From φ to **E** $\vec{E} = -\nabla \phi$
  - One derivative is all it takes but... make sure you choose the best coordinate system
  - You will not loose points but you will waste time...
- From φ to ρ
  - Poisson tells you how to get from potential to charge distributions directly:

$$\nabla^2 \phi = -4\pi\rho$$

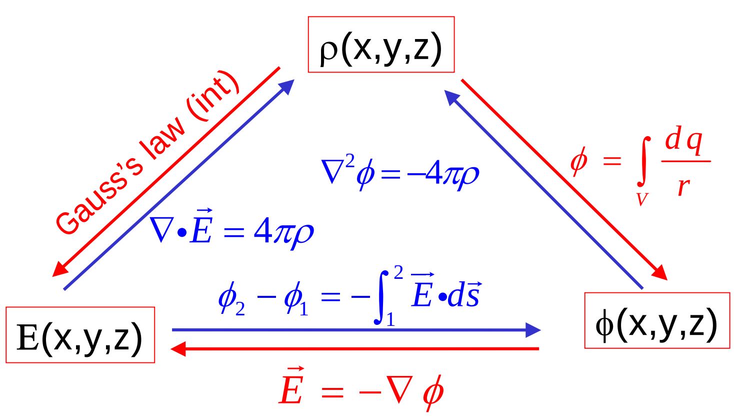
  - Uncomfortable with Laplacian? Get there in 2 steps:
    - First calculate E: $\vec{E} = -\nabla \phi$
    - The use differential form of Gauss's law: $\nabla \bullet \vec{E} = 4\pi\rho$

Thoughts about ϕ and **E**

- The potential ϕ is always continuous
- E is not always continuous: it can "jump"
  - When we have surface charge distributions
    - Remember problem #1 in Pset 2

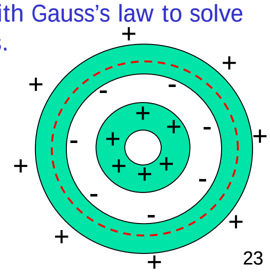
→ When solving problems always check for consistency!

Summary



Conductors

- Properties:
  - Surface of conductors are equipotential
  - E (field lines) always perpendicular to the surface
  - $E_{inside}=0$
  - $E_{surface}=4\pi\sigma$
- What's the most useful info?
  - $E_{inside}=0$ because it comes handy in conjunction with Gauss's law to solve problems of charge distributions inside conductors.
  - Example: concentric cylindrical shells
    - Charge +Q deposited in inner shell
    - No charge deposited on external shell
    - What is E between the 2 shells?
      - - Q induced on inner surface of inner cylinder
      - +Q induced on outer surface of outer cylinder

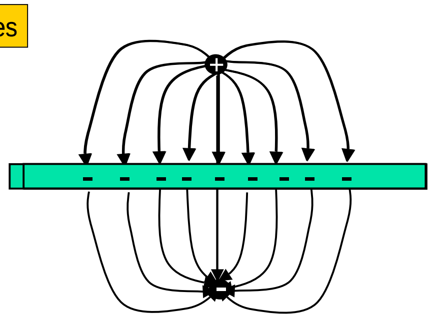


E due to Charges and Conductors

- How to find E created by charges near conductors?
  - Uniqueness theorem:
    - A solution that satisfies boundary conditions is THE solution
  - Be creative and think of distribution of point charges that will create the same filed lines:

Method of images

  - Example:



Capacitors

- Capacitance
  - Two oppositely charged conductors kept at a potential difference V will have capacitance C

$$C = \frac{Q}{V}$$

  - NB: capacitance depends only on the geometry!
- Energy stored in capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

- What should you remember?
  - Parallel plate capacitor: very well
  - Be able to derive the other standard geometries

Conclusion

- Material for Quiz #1:
  - Up to this lecture (Purcell chapters 1/2/3)
- Next lecture:
  - Charges in motion: currents
  - NB: currents are not included in Quiz 1!