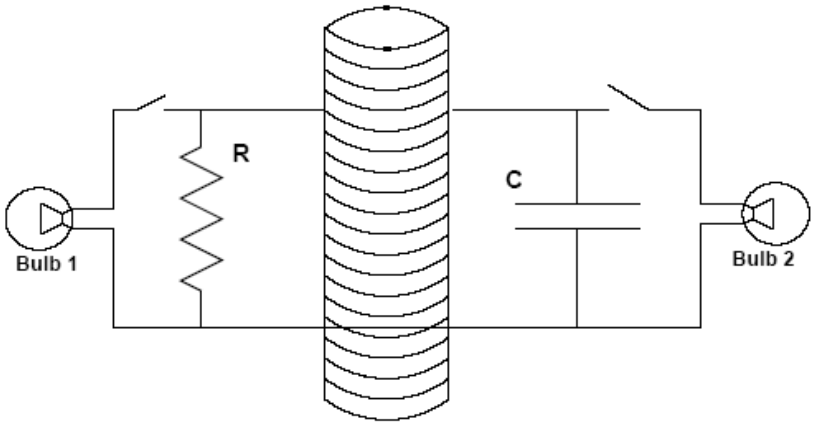
Pset 9, Problem 7 (instead of Purcell 9.X)

NB: the solenoid is positioned in such a way that its axis is perpendicular to the page and to the plane of the circuit. Not obvious from the picture…

An infinite solenoid is enclosed by a series RC circuit. The solenoid has a radius b and N turns of wire per unit length. The resistor has resistance R and the capacitor has capacitance C. By closing a pair of switches, we can put a pair of lightbulbs in parallel with the resistor and the capacitor. In what follows, the solenoid is resistanceless and we may ignore the self inductance of the circuit around the solenoid.



The current through the solenoid is turned on at $t = 0$ and from that time has the form $I_{\text{sol}}(t) = \alpha t$; $\alpha > 0$. This current cycles counterclockwise (indicated by arrows). Both switches are open.

(a) What EMF $\mathcal{E}$ is induced in the RC circuit?

(b) Compute the charge across the capacitor, $Q(t)$, and the current through the resistor, $I(t)$, as functions of time. Which plate of the capacitor is positive and which is negative?

(c) Suppose that when we turn the solenoid on, the switches are both closed. Describe qualitatively the brightness of the bulbs as a function of time. (Assume the resistance of the bulbs is high enough that you do not need to worry about how the current through those bulbs will change your solution. The brightness is then proportional to the squared voltage in the bulb's circuit element.)

The current through the solenoid is now given by $I = I_0 \sin(\omega t)$ ($I_0 > 0$). The lightbulb switches are both closed, so that the lightbulbs are part of the circuit.

(d) Working in the complex representation, find the voltage across the resistor $\tilde{V}_R$ and the voltage across the capacitor $\tilde{V}_C$.

(e) Describe qualitatively the brightness of the bulbs as the frequency is varied from $\omega = 0$ to $\omega = \infty$.

(e) At what frequency is the brightness of the bulbs exactly equal?

(f) As the oscillation frequency is made very large, which bulb is most likely to burn out?