

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

Physics 8.022

Spring 2003

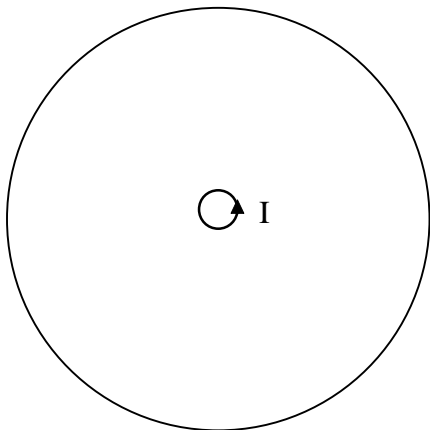
ELECTRICITY AND MAGNETISM
QUIZ #3 from last year

Problems on the final will be similar to those found here – over 50% of the problems on the final will focus on material covered since the last quiz. For practice problems from earlier in the course, please refer back to practice exams 1 and 2 and the homework.

There are five questions. Answer all of the questions. This is a closed book quiz

Problem #1 (Magnetic Fluxes) (15 pts.)

Find the total net magnetic flux from the small loop shown below that threads the large loop. The two loops have precisely the same center. The large loop has a radius b and the small loop has a radius a . You should consider $a \ll b$. Hint: this problem does not involve doing any complicated integrals.

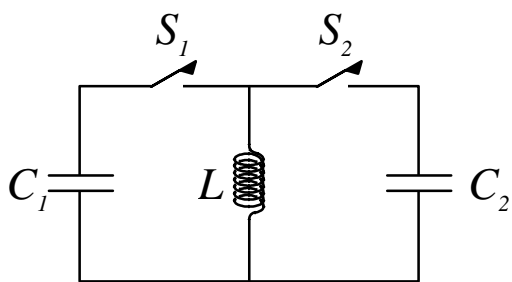


Problem #2 (An Electromagnetic Wave) (10pts)

If the electric field in free space is $\vec{E} = \frac{E_0}{\sqrt{2}} \hat{x} \sin(kx + \omega t) + \frac{E_0}{\sqrt{2}} \hat{y} \sin(kx + \omega t)$. There is corresponding a magnetic field that also varies with frequency ω . What is this field?

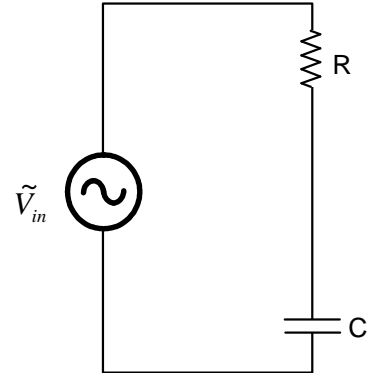
Problem #3 (Two capacitors and an inductor) (20pts)

Consider the following circuit where C_1 is initially charged to 75 V. Suppose that C_1 is $10,000 \mu\text{F}$, C_2 is $3000 \mu\text{F}$, and L is 15 H . Explain how to open and close the switches so as to discharge C_1 and charge C_2 . Starting at $t = 0$, you should give explicitly times for opening and closing each switch. What is the final voltage across C_2 ?



Problem #4 (Drawing current from a complex impedance) (30 pts)

Consider the circuit on the right containing a resistor and a capacitor in series and driven by an ac voltage source operating at frequency ω . Throughout this problem, leave your answers in complex form (e.g. \tilde{I} , \tilde{V}).



a) What is the complex current through this circuit?

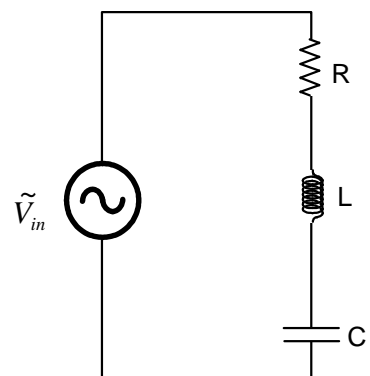
In your derivation, you may use complex numbers (including terms involving the imaginary number i), but you should rationalize your final answer to express it in the form $\tilde{I} = f(\omega, R, C, L)e^{i\phi}\tilde{V}_{in}$ where

$f(\omega, R, C, L)$ is a real function of the frequency, resistance, capacitance, and inductance in the circuit. Also, give an expression for ϕ . NOTE: All answers to each part of this problem (problem 4 parts a-h, should be given in this format – as a real function multiplied by a phase factor).

Now, modify this circuit by adding in an inductor. Also, consider it to be driven at its resonant

frequency, $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$.

b) What is the complex current \tilde{I} that flows through the circuit if we consider the source voltage as complex \tilde{V}_{in} . Remember, the source voltage is oscillating at the resonant frequency for this circuit.



You can just write down the answer with no derivation. (HINT: at resonance, what is the sum of the impedances of the inductor and the capacitor?)

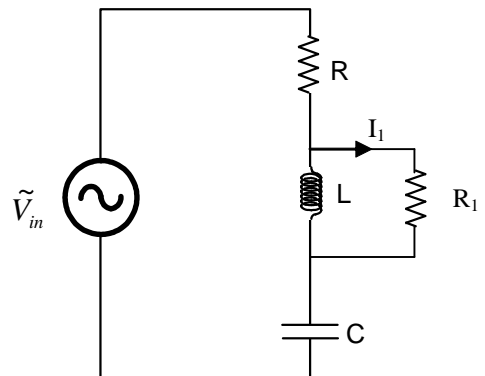
c) What is the complex voltage across the inductor, \tilde{V}_L ? Express it as a real quantity multiplied by \tilde{V}_{in} and a complex phase factor $e^{i\theta}$. As much as possible, eliminate any terms involving L , R , and C from your answer and replace them with Q ($Q = \frac{\omega_0 L}{R}$).

d) One might consider this voltage across the inductor as a source that can be used to drive currents. You then have Thèvenin equivalent voltage and equivalent source impedance. To determine the Thèvenin equivalent impedance, we first need to know the complex current \tilde{I}_{sc} that would flow through a wire short-circuiting the inductor. Determine \tilde{I}_{sc} . (HINT: Given the work you've already done, this part should be quite easy).

e) Determine the Thévenin equivalent impedance (complex) $Z_{Thev} = \frac{\tilde{V}_{OC}}{\tilde{I}_{SC}}$. Write your answer in terms of a real quantity and a phase factor $e^{i\alpha}$. Express α in terms of Q and R (there should be no L 's or C 's in your answer).

f) Simplify your answer from (e) for the limit where $Q \gg 1$ and express your answer using terms of Q and R (there should be no L 's or C 's in your final answer). What happens to the phase factor $e^{i\alpha}$ as $Q \rightarrow \infty$?

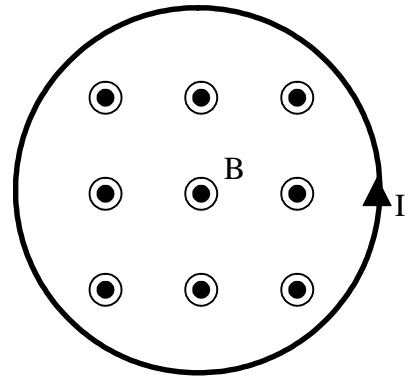
g) Now, consider the circuit on the right, again in the limit of large Q and excited at the resonant frequency. What is the amplitude and phase of the current \tilde{I}_1 that flows through the resistor R_1 ?



h) If the exciting voltage is \tilde{V}_{in} represents a voltage $V_{in} \cos(\omega t)$, what is the actual current I_1 in the limit of large Q ?

Problem #5 (Poynting Vector in a Solenoid) (25pts)

In the figure shown to the right, you are looking down the barrel of a solenoid. The solenoid is oriented so that a current I flowing in the coil produces a magnetic field pointing out of the page. Everywhere in this problem, you may use the assumption that the solenoid behaves as an infinite solenoid with infinitely tight windings.



- a) If the current running through the solenoid's coil is increasing at a constant rate, there will be an electric field inside the solenoid. Draw arrows in the figure above to represent the direction of this electric field.
- b) If we call the total flux in the solenoid Φ_T , what is the magnitude of the electric field in terms of $\frac{d\Phi_T}{dt}$? Let r be the distance from the central axis of the solenoid to the point of interest. The radius of the solenoid is a .
- c) The solenoid has a length l . Use the Poynting vector $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$ to determine the total power flowing into the solenoid. Express your answer in terms of $\Phi_T, \frac{d\Phi_T}{dt}, a, l$.

d) The solenoid has N turns (total) is a length l long and has a radius a . What is the inductance of the solenoid? Use Ampère's law to determine the field of a solenoid if you don't remember it.

e) Using the formula $Power = LI \frac{dI}{dt}$, determine the power required to increase the flux in the solenoid. Express your answer in the same terms as in part (c). HINT: The two answers had better agree!