Massachusetts Institute of Technology
Department of Physics
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Useful Formulae

Conservative Field: $\int_C \vec{F} \cdot d\vec{r} = 0$ for any closed path C, $W_{ab,C} = W_{ab,C'}$ for any C, C' connecting a *and* b, $\vec{F} = -\vec{\nabla} U$, $\vec{\nabla} \times \vec{F} = 0$

Coulomb Law: $\vec{F}_{21} = \frac{q_1 q_2}{r^2} \hat{r}_{21}$ for two point charges at distance r. $\vec{F}_{12} = -\vec{F}_{21}$, and for charges dq_1 and dq_2 that make part of continuous charge distributions 1 and 2, $d\vec{F}_{21} = \frac{dq_1 dq_2}{r^2} \hat{r}_{21}$

Electric Field: *at point 2* due to q_1 $\vec{E}_1 = \frac{q_1}{r^2} \hat{r}_{21}$. If q_1 is not a point charge but part of a continuous distribution, $d\vec{E} = \frac{dq}{r^2} \hat{r}$

Principle of Superposition: Two or more electric fields acting at a given point P add vectorially: $\vec{E}_P = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$

Electrostatic Field is Conservative: $\vec{\nabla} \times \vec{E} = 0$ and thus there exists scalar function ϕ such that $\vec{E} = -\vec{\nabla} \phi$ where $d\phi = \frac{dq}{r}$

Electrostatic potential: The potential at $\vec{x}$ with respect to a *ref* point is $\phi(\vec{x}) - \phi(ref) = -\int_{ref}^{\vec{x}} \vec{E} \cdot d\vec{r} = -\frac{W_{ref \to \vec{x}}}{q}$

Gauss Law: $\int_S \vec{E} \cdot d\vec{a} = 4\pi \int_V \rho dV$ where S is a closed surface and V is its corresponding volume (integral form) or $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ (differential form).

Poisson Eqn: $\nabla^2 \phi = -4\pi\rho$, Laplace Eqn: $\nabla^2 \phi = 0$

Energy: $U = \frac{1}{2} \int_V dV \int_{V'} dV' \frac{\rho(\vec{x})\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} = \frac{1}{2} \int_V \rho \phi dV = \frac{1}{8\pi} \int_V E^2 dV$

Electric Force on Conductors: $\frac{dF}{da} = 2\pi\sigma^2 = \frac{E^2}{8\pi}$

Current Density: $\vec{J}(\vec{x}) = \rho(\vec{x})\vec{v}(\vec{x})$, Conservation Law/Continuity: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

Currents: $I = \frac{dq}{dt}, I = \int_S \vec{J} \cdot d\vec{a}$

Capacitance: $Q = CV$, (energy) $U = \frac{1}{2} CV^2$

Capacitor networks: Parallel: $C = C_1 + C_2$, series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Ohm's Law: $\vec{J} = \sigma \vec{E}, V = IR$

Resistor networks: Series: $R = R_1 + R_2$, parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Magnetic charges: $\vec{\nabla} \cdot \vec{B} = 0$

Biot-Savart's Law: $d\vec{B} = \frac{I d\vec{l} \times \hat{r}}{cr^2}$

Lorentz Force: $\vec{F} = q\vec{E} + \frac{q}{c} \vec{v} \times \vec{B}$

Force on current: $\vec{F} = \frac{I}{c} d\vec{l} \times \vec{B}$

Ampere's Law: $\oint_C \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_{encl} = \frac{4\pi}{c} \int_S \vec{J} \cdot d\vec{a}$, $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J}$

Relativistic Transformations:
All primed quantities measured in the frame F' which is moving in the positive x direction with velocity $u = \beta c$ (and $\gamma = 1/\sqrt{1-\beta^2}$) as seen from F.
Position and momentum 4-vectors:

$$x' = \gamma(x - \beta ct) \quad p' = \gamma(p - \beta \tfrac{E}{c})$$
$$t' = \gamma(t - \beta \tfrac{x}{c}) \quad E' = \gamma(E - \beta cp)$$

Electric and magnetic field components:

$$E'_x = E_x \quad E'_y = \gamma(E_y - \beta B_z) \quad E'_z = \gamma(E_z + \beta B_y)$$
$$B'_x = B_x \quad B'_y = \gamma(B_y + \beta E_z) \quad B'_z = \gamma(B_z - \beta E_y)$$

Relativistic Mass, Energy: $m = \gamma m_0, E = mc^2$

Faraday's Law: $\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt}, \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$

Mutual Inductance: $M_{12} = M_{21} = \frac{\Phi_{21}}{cI_1}, \mathcal{E}_{21} = -M_{21}\frac{dI_1}{dt}$

Self Inductance: $L = \frac{\Phi}{cI}, \mathcal{E} = -L\frac{dI}{dt}$

Magnetic Field Energy Density: $\frac{dU_B}{dv} = u_B = \frac{B^2}{8\pi}$

Impedance: $V = ZI, Z_R = R, Z_C = -\frac{i}{\omega C}, Z_L = i\omega L$

Admittance: $Y = 1/Z$

Complex notation: $e^{i\theta} = cos\theta + isin\theta, z = \alpha + ib = |z|e^{i\theta}, |z| = \sqrt{\alpha^2 + b^2}, tan\theta = b/\alpha$

Displacement: current $I_d = \frac{1}{4\pi}\frac{d\Phi_E}{dt}$, density $\vec{J}_D = \frac{1}{4\pi}\frac{\partial \vec{E}}{\partial t}$

Maxwell's equations: $\vec{\nabla} \cdot \vec{E} = 4\pi\rho, \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

Electromagnetic waves: $\vec{E} = \vec{E}_0 f(\vec{k}\vec{r} - \omega t), \vec{B} = \vec{B}_0 f(\vec{k}\vec{r} - \omega t), |\vec{E}| = |\vec{B}|, k = 2\pi/\lambda$, $\omega = 2\pi/T, c = \omega/k, \vec{E} \times \vec{B}$ points along propagation direction $(\hat{k})$

Wave "mechanics": Poynting vector $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$, momentum density $\vec{g} = \frac{1}{4\pi c}\vec{E} \times \vec{B}$, energy density $u_T = \frac{E^2}{8\pi} + \frac{B^2}{8\pi}$, radiation pressure $\frac{F}{A} = u_T$

Waveguides: $\frac{\partial I}{\partial x} = -C_0\frac{\partial V}{\partial t}, \frac{\partial V}{\partial x} = -L_0\frac{\partial I}{\partial t}, \frac{\partial^2 I}{\partial x^2} - L_0 C_0 \frac{\partial^2 I}{\partial t^2} = 0, \frac{\partial^2 V}{\partial x^2} - L_0 C_0 \frac{\partial^2 V}{\partial t^2} = 0$

Gradient: in cartesian $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$, in cylindrical $\vec{\nabla} f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$, in spherical $\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$

Divergence: in cartesian $\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$, in cylindrical $\vec{\nabla} \cdot \vec{F} = \frac{F_\rho}{\rho} + \frac{\partial F_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$, in spherical $\vec{\nabla} \cdot \vec{F} = \frac{2F_r}{r} + \frac{\partial F_r}{\partial r} + \frac{F_\theta}{r}cot\theta + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{1}{rsin\theta}\frac{\partial F_\phi}{\partial \phi}$