8.022 (E&M) – Lecture 22

Topics:

- Magnetic properties of materials
  - Magnetic dipole of electrons → macroscopic behavior of matter
  - Properties of Diamagnetic, Paramagnetic and Ferromagnetic materials

Final Exam

- When and where?
  - Tue Dec. 14, 9:00 – 11:00 AM
    - Please arrive 10 min early: no extra time given if you are late!
- Format of the exam
  - Similar to quiz 1 and 2: 4 problems, 2 hours
    - Same difficulty, more time: you must do a better job!
  - Topics
    - 1 problem on Quiz 1 material (Electrostatics,…)
    - 1 problem on Quiz 2 material (Currents, Relativity, Induction,…)
    - 2 problems on post-Quiz 2 material (RCL, AC circuits, waves,…)

Final: FAQ

- When will the final be graded?
  - Immediately after the exam: by 6 PM Tue we expect to be done
- What is the passing grade?
  - Freshmen: C
  - Everybody else: D
- What is the passing score?
  - We have not decided yet. It depends on how hard the final will be
  - Be assured you will be graded fairly and consistently.

How to prepare?

- Read and understand all lecture notes
  - Fast and (hopefully) easy to digest. (Almost) All you need to know
- Go back to section notes
  - Solve problems done in class by yourselves and check answers
- Go back to homework problems
  - Solve them again and compare answers
- Solve old exam problems (posted on Tue)
  - Useful to understand how fast you can solve the problems
- Attend review session on Sat
  - And office hours if you have last minute questions
- Read Purcell
  - If you have time left. You should have done this already…

How to simplify your life (and ours):

A few tips to a high score

- Remember: 35 points assigned to final
  - Quiz 1: 20 points, Quiz 2: 20 points, Make up: 7 points
- Sleep at least 6 hours the night before
  - Being able to THINK is your most important asset!
- Not sure how to interpret a question/figure? ASK!!!
  - That's why we are in the exam room!
- Read all the problems and start working on what you know best
  - Don't spend 80% of your time on the one problem you cannot solve: 3 perfect problems will give you 75 points
  - Partial credit: if you are unable to solve part a) see if you are able to solve b)
- Make sure you answer ALL the questions:
  - When you are done, go back to the text and make sure answers are complete (vector direction, etc)

Back to physics…

- Last time: end of 8.022 official program
  - Energy and momentum carried by EM waves
    - Poynting vector and some of its applications
  - Transmission lines
  - Scattering of light through matter

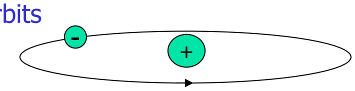
- Today: beyond scope of 8.022, just enjoy!
  - Magnetic properties of materials
    - Where do they come from?

Magnetic properties of materials

- We went through the whole E&M course without even understanding how a magnet bar works?
  - Yes, so far. Let's try to make up for this… ☺
  - Very qualitative description: as far as we can go without quantum since

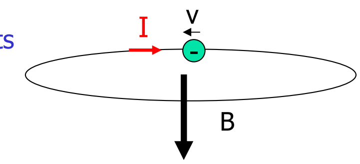
> Magnetic properties of materials are totally determined by quantum mechanical nature of their molecular structure

- In the discussion I assume we are all familiar with some basic properties of electrons and atoms
  - Nucleus at the center, electrons rotating on orbits
  - Electron is negatively charged
  - Electron has an intrinsic angular momentum (spin)

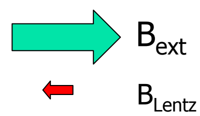


Effects due to electron orbits

- Electrons in atoms produce magnetic field
  - Electrons rotate around the nucleus in orbits
  - This is same as having a loop of current
  - Currents produce magnetic fields (Ampere)



- It's usually a small effect…
  - There are lots of electrons, orbits are randomly oriented: cancellation
- What happens when we put the material in an external B?
  - Lentz's law: the orbits rearrange so that the magnetic field created by the orbits opposes the external magnetic field
- Net effect: the total magnetic field will be weaker



Magnetic moments of electrons

- Current due to electron in orbit of radius r: $I = \frac{ev}{2\pi r}$
- The magnetic moment μ of the loop is

$$\mu \equiv \frac{IA}{c} = \frac{\pi r^2 I}{c} = \frac{evr}{2c}$$

- The magnetic moment μ is related to the angular momentum L:

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \boxed{\vec{\mu} = \frac{-e\vec{L}}{2m_e c}}$$

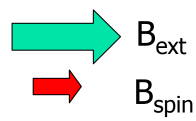
- In addition to the standard angular momentum L electrons have intrinsic angular momentum (spin) → intrinsic magnetic moment
  - Will this contribute to macroscopic magnetic properties of material?

Effects due to electron's spin

- The intrinsic magnetic moment behaves very differently from the standard magnetic moment
  - No Lentz's law type behavior because this field is associated with the electron itself
- What happens when we put the material in an external B?
  - A magnetic moment μ placed in an external filed B feels a torque
    - See Purcell 6.22

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

  - τ tends to line up the electron magnetic moments with external field
- Net effect: the total magnetic field will be stronger



What effect is stronger?

- Summary of the situation so far:
  - Lentz's law on the orbit of the electrons opposes B fields from entering material
  - Magnetic torque acting on individual electrons augments the B field in the material
- Opposite behaviors! Who wins?
  - It depends on the properties of the material (chemical structure, how free electrons are, etc)
- 3 categories:
  - Diamagnetic materials
  - Paramagnetic materials
  - Ferromagnetic materials

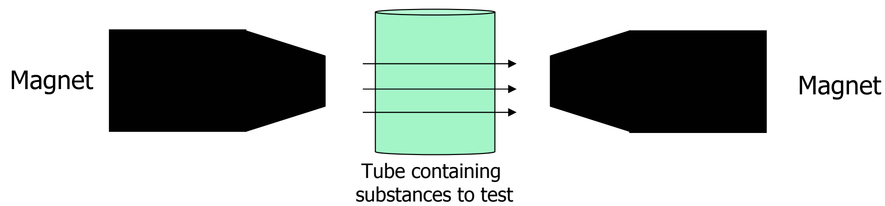
Diamagnetic materials

- Diamagnetic materials defined as materials in which the magnetization opposes the external magnetic field
  - When material is immersed in external B field, magnetic field inside the material is weaker than external B
  - Lentz's law wins out on effect of spin
- Diamagnetism is usually very weak and hard to see
  - Lentz's law plays a role in <u>all</u> materials. Spin effect (if present) are stronger → if preset it usually covers completely diamagnetic behavior
- Examples of diamagnetic materials
  - Typically orbits filled with paired electron → orbit has no net magnetic moment
  - Most substances: H_20, Cu, NaCl, etc
- Consequence: diamagnetic substances will be expelled from B field

Paramagnetic materials

- Paramagnetic materials are defined as materials in which the magnetization augments the external magnetic field
  - When material is immersed in B field, magnetic field inside the material is stronger than outside
  - Effect of Spin wins out on Lentz's law
- Examples of diamagnetic materials
  - Typically have several electron orbits that contain unpaired electrons → orbit has a net magnetic moment
    - Exception: Oxygen O_2 is paramagnetic. To see this property need to cool it to a liquid state, or random motion will wipe out effect
  - Example: Na, Al, $NiSO_4$, etc
- Consequence: Paramagnetic materials are pulled into magnetic fields
- If paramagnetic behavior is "extra strong": Ferromagnetic material

Diamagnetic and paramagnetic materials (J4 and J6)



- What happen when we put bismuth in the tube?
  - Remember: Bismuth is diamagnetic!
- What if we put Al?
  - Al is paramagnetic → orients // to B
- What happen if we pour liquid O_2 between magnets? What if we pour H_2O?
  - Remember: O_2 liquid is paramagnetic, water is diamagnetic

Magnetization and H field

- Magnetization M is defined as the magnetic dipole moment of a substance per unit volume
  - Magnetic moment of a material with volume V and magnetization M $\vec{\mu}=\vec{M}V$
  - Dimension analysis

$$[\vec{M}]=\frac{\text{magnetic moment}}{\text{volume}}=\frac{\text{current} \times \text{area/velocity}}{\text{volume}}=\frac{\text{current /velocity}}{\text{length}}=[B]$$

  → M has same dimensions as magnetic field B

- Define a new kind of magnetic field H

$$\vec{B}=\vec{H}+4\pi\vec{M}$$

  - B is the total magnetic field; H is the "normal field" due to currents, M is the magnetization, component of B due to material's properties
  - In vacuum, B=H

H and Maxwell's equations

- Let's quickly look at something that you will study in 8.07
  - The curl of H defines the free electrical currents

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}_{free} \text{ with } \vec{J}_{free} = \text{density of free electrical current}$$

  - The curl of the magnetization defines the bound currents

$$\nabla \times \vec{M} = \frac{1}{c}\vec{J}_{bound} \text{ with } \vec{J}_{bound} = \text{density of bound electrical current}$$

- Plug into Ampere's law

$$\nabla \times \vec{B} = \nabla \times \vec{H} + 4\pi\nabla \times \vec{M} = \frac{4\pi}{c}\left(\vec{J}_{free} + \vec{J}_{bound}\right) = \frac{4\pi}{c}\vec{J}$$

$$\Rightarrow \boxed{\vec{J} = \vec{J}_{free} + \vec{J}_{bound}}$$

  - Total current density is due to sum of current that we can control J_{free} and current due to the material J_{bound}

Magnetic susceptibility

- Many substances exhibit linear magnetization, e.g. the magnetization depends linearly on the external field applied

$$\vec{H} = \chi_m \vec{M}$$

  - Where χ_m= magnetic susceptibility
- Since $\vec{B} = \vec{H} + 4\pi\vec{M}$ it follows that

$$\vec{B} = \vec{H}(1 + 4\pi\chi_m)$$

- Classification of material based on magnetic susceptibility
  - χ_m<0: magnetic field B decreases when we immerse the substance in an external magnetic field B: diamagnetism
  - χ_m>0: magnetic field B increases when we immerse the substance in an external magnetic field B: paramagnetism

Magnetic properties of materials

Classification of materials based on magnetic susceptibility

- Paramagnetic: $\chi_m > 0$
- Diamagnetic: $\chi_m < 0$

Material	$\chi_m (10^{-5})$
Uranium	40
Aluminum	2.2
Oxygen gas	0.2
$H_2 0$	-0.9
Lead (Pb)	-1.8
Carbon (diamond)	-2.1
Bismuth	-16.6

Ferromagnetism

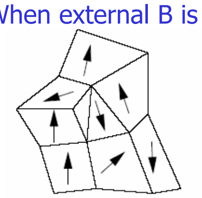
- "Ferromagnetism is paramagnetism on steroids"

  Prof. S. Hughes – 8.022 S-2004

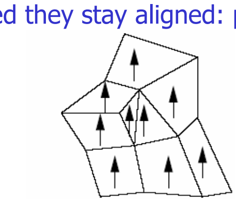
- Nonlinearity distinguishes it from paramagnetism
  - M and H do not have a simple linear relation
  - Magnetization remains after external field is turned off
    - This is how permanent magnets work!
- Why nonlinear behavior?
  - Way beyond scope of 8.022
  - But it's easy enough to describe qualitatively how it works…

Ferromagnetic domains

- Ferromagnetism is conceptually similar to paramagnetism
- Difference: magnetic moments of many atoms are tend to be aligned in small regions (domains)
  - Paramagnetic materials: moments are randomly arranged until external B aligns them
  - Since domains are small (0.1 mm – few mm) and randomly oriented: overall M=0
- When material is put into external B, domains re-align // to B
  - When external B is removed they stay aligned: permanent magnets!



Ferromagnetic material before B is applied



Ferromagnetic material after B is applied

Curiosity: you cannot see domains flipping but you can hear them: Barkhausen effect (J3)

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Sizes of domains range from a 0.1 mm to a few mm. When an external magnetic field is applied, the domains already aligned in the direction of this field grow at the expense of their neighbors. If all the spins were aligned in a piece of iron, the field would be about 2.1 Tesla. A magnetic field of about 1 T can be produced in annealed iron with an external field of about 0.0002 T, a multiplication of the external field by a factor of 5000!

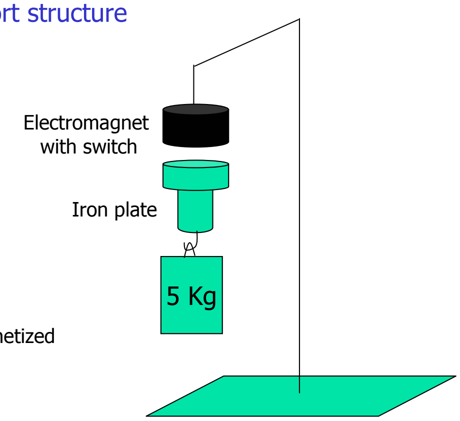
Barkhausen effect:

Domains are well modeled by the compass table, an array of about one hundred small compass needles used for showing fields of bar magnets, etc. When there is no strong external Bfield, sections of the array line up in different directions, each individual compass needle aligning itself with the local field. When the array is tapped sharply, it will be seen that the needles on the boundaries of the domains are the least stable (vibrate the most), and some of them realign causing one domain to grow at the expense of another.

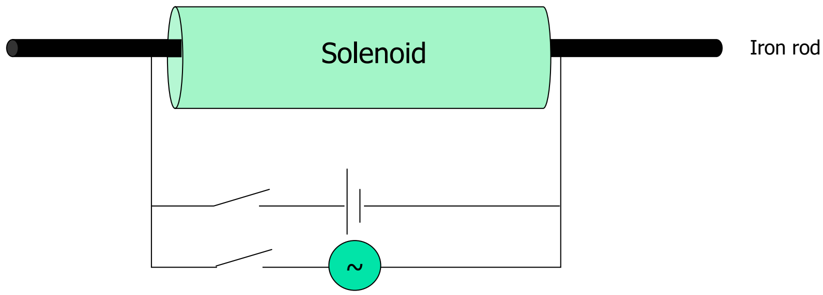
In the Barkhausen effect, a large coil of fine wire is connected through an amplifier to a speaker. When an iron rod is placed within the coil and stroked with a magnet, an audible roaring sound will be produced from the sudden realignments of the magnetic domains within the rod. A copper rod, on the other hand, produces no effect.

Magnetization of iron (J2)

- Electromagnet is hanging from a support structure
  - Initially off
    - Fe plate falls off
- Turn on electromagnet
  - Domains line up
    - Fe plate will stick
- Turn off electromagnet
  - Domains still lined up
    - Fe plate will stick because it's now magnetized
    - Add up to 5 Kg or so!
- When it falls: domains break
  - Fe plate will fall



Magnetization and demagnetization of iron rod (J7)

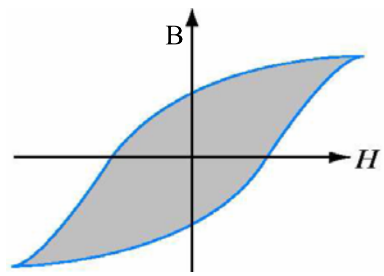


- Use DC to magnetize iron rod
  - Current in solenoid creates B; Fe rod's domains align with B

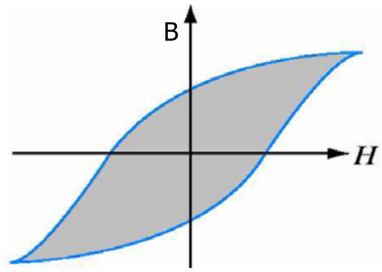
    → rod becomes magnetic: it attracts paper clips etc
- Demagnetize Fe rod with AC current
  - Run it slower and slower, flipping the direction of the domains slower and slower

Nonlinearity and hysteresis

- In ferromagnetic materials B and H have a nonlinear dependence
  - Let's find out experimentally what that is
    - Apply external field H (x axis) and measure total field B (y axis) in the ferromagnetic material
    - Start with value of H (H_0), decrease to 0, flip the direction and reach $-H_0$
- The curve describing relationship between H and B is called hysteresis curve
  - When H=0, B.ne.0
  - What value will it take? +H? –H?
  - It depends on the magnetization history



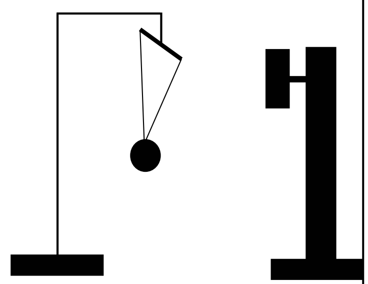
Histeresis curve of Fe core transformer (J11)



- Measure hysteresis curve using the Fe core of a transformer
  - Send to an oscilloscope:
    - Channel 1 (x): I into primary winding of the transformer (H)
    - Channel 2 (y): I from secondary winding (B)

Curie Temperature

- Curie temperature is the temperature above which ferromagnetic materials stop acting as such
  - NB: transition is very sudden!
- Why does it happen?
  - At $T>T_C$ the random motion of the magnetic moments becomes so strong that they cannot align anymore to form domains
  - For Fe: T_C=770° C
- Demo J10
  - Iron nut sticks to permanent magnet
  - Heat up the nut until it reaches T_C
    - Nut will temporarily loose its ferromagnetic properties.



Summary

- Today:
  - Magnetic properties of materials
    - Diamagnetism, paramagnetism, ferromagnetism
  - Qualitative description
    - Quantum will answer deeper questions: SOON!!! ☺

- Final Exam:
  - On Tue Dec. 14, 9:00 – 11:00 AM
  - Please arrive 10 min early!

If you enjoyed 8.022, you will love Quantum Mechanics!
Physics is cool!

Merry Christmas… the 8.022 way!

$$
\begin{cases}
\vec{\nabla} \bullet \vec{E} = 4\pi\rho \\
\vec{\nabla} \bullet \vec{B} = 0 \\
\vec{\nabla} \times \vec{E} = -\dfrac{1}{c}\dfrac{\partial \vec{B}}{\partial t} \\
\vec{\nabla} \times \vec{B} = \dfrac{4\pi}{c}\vec{J} + \dfrac{1}{c}\dfrac{\partial \vec{E}}{\partial t}
\end{cases}
$$