Class 18: Outline

Hour 1:

Levitation

Experiment 8: Magnetic Forces

Hour 2:

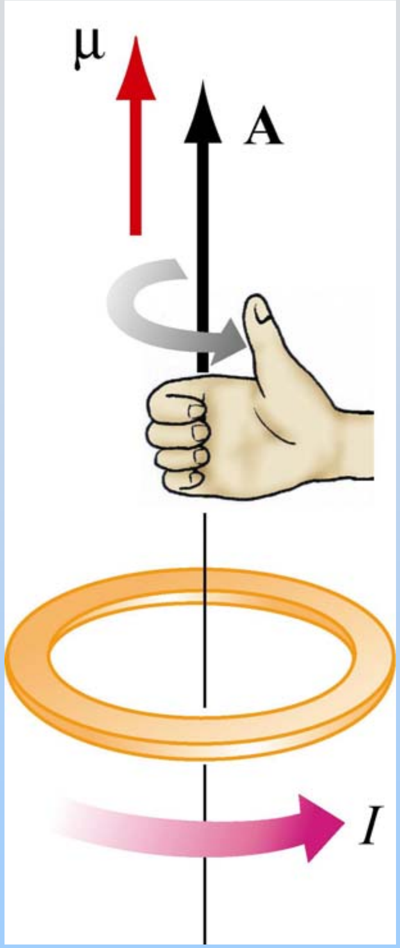
Ampere's Law

Review:
Right Hand Rules

1. Torque: Thumb = torque, fingers show rotation
2. Feel: Thumb = I, Fingers = B, Palm = F
3. Create: Thumb = I, Fingers (curl) = B
4. Moment: Fingers (curl) = I, Thumb = Moment

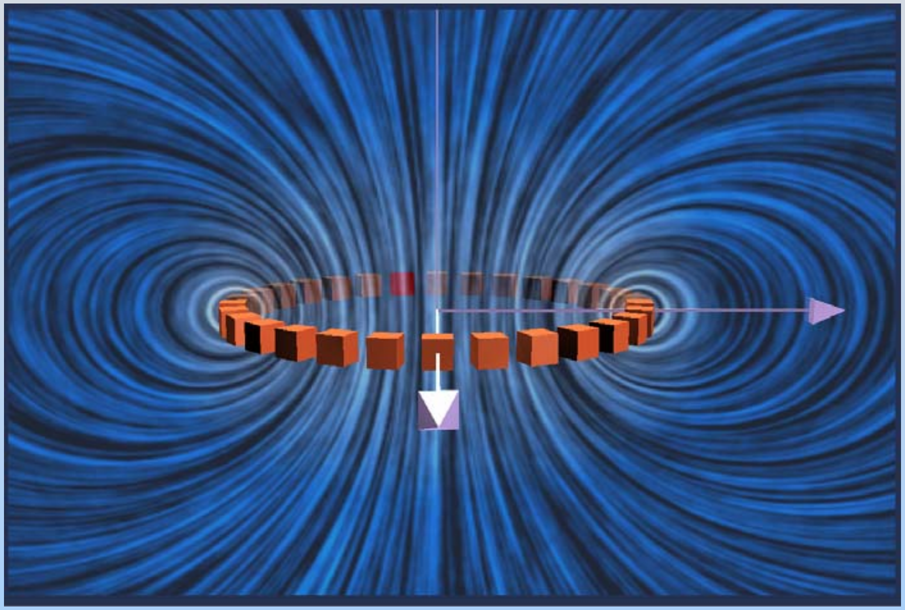
Last Time:
Dipoles

Magnetic Dipole Moments



$$\vec{\boldsymbol{\mu}} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

<u>Generate:</u>



<u>Feel:</u> $U_{Dipole} = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

1) Torque to align with external field
2) Forces as for bar magnets (seek field)

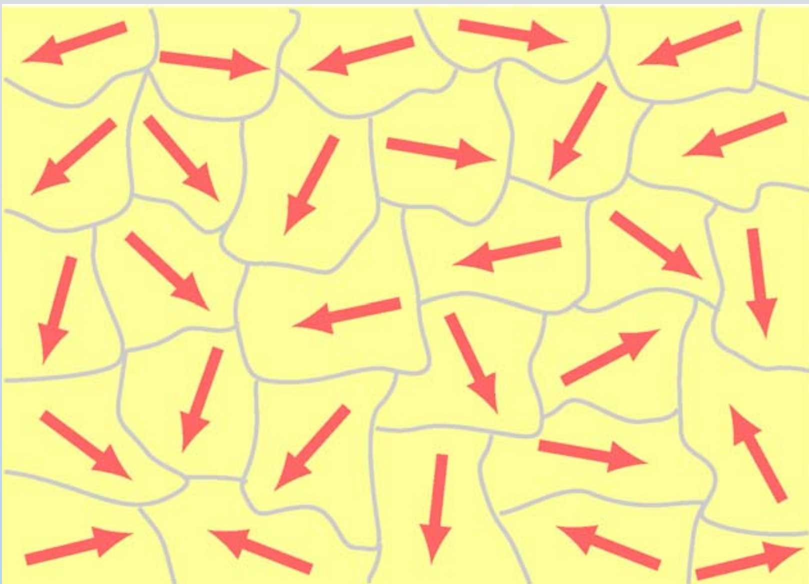
Some Fun:
Magnetic Levitation

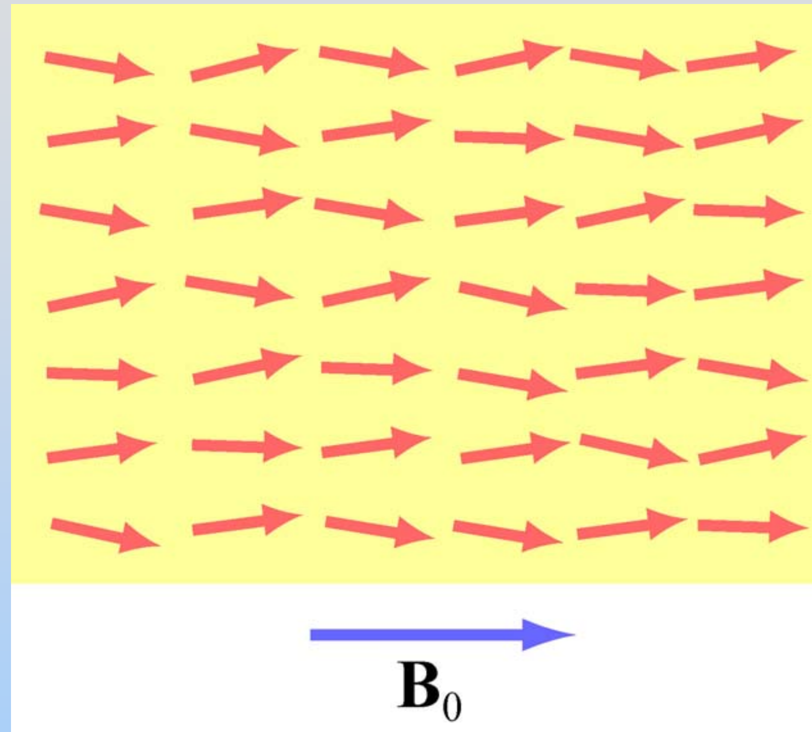
Put a Frog in a 16 T Magnet…

For details: http://www.hfml.sci.kun.nl/levitate.html

How does that work?
First a BRIEF intro to
magnetic materials

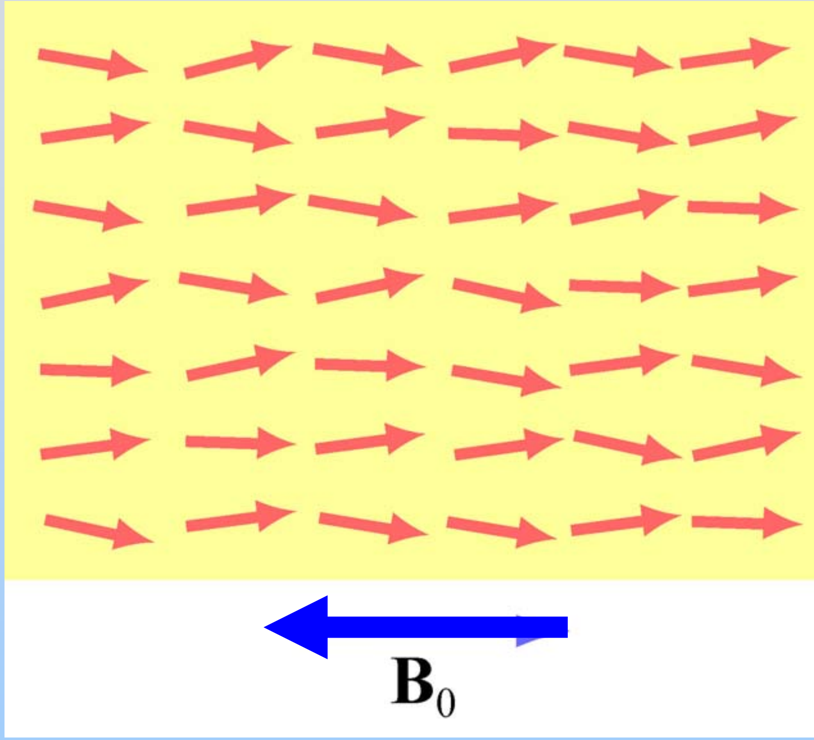
Para/Ferromagnetism





Applied external field B_0 tends to align the atomic magnetic moments (unpaired electrons)

Diamagnetism

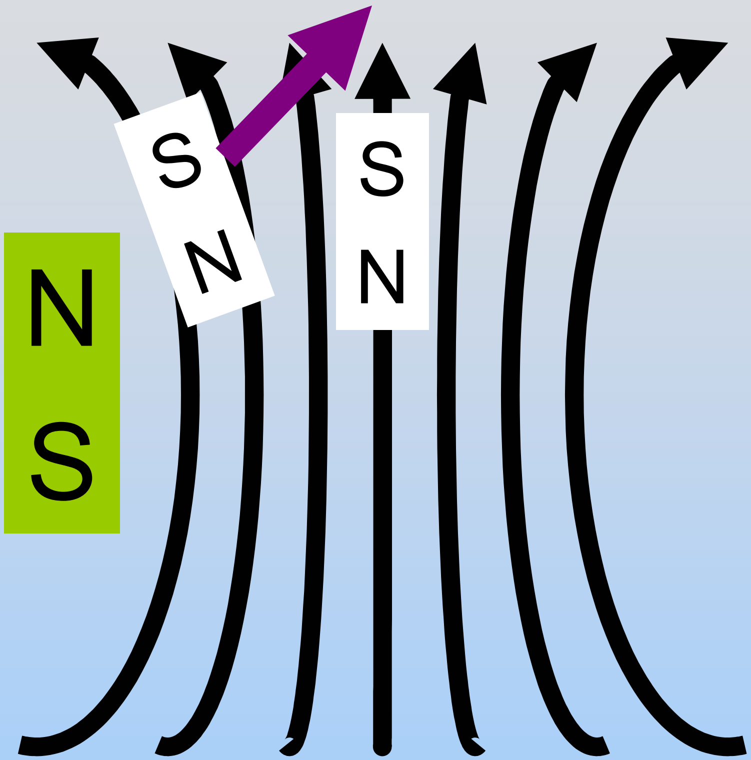


Everything is slightly diamagnetic. Why? More later.

If no magnetic moments (unpaired electrons) then this effect dominates.

Back to Levitation

Levitating a Diamagnet



1) Create a strong field (with a field gradient!)
2) Looks like a dipole field
3) Toss in a frog (diamagnet)
4) Looks like a bar magnet pointing *opposite* the field
5) Seeks *lower* field (force *up*) which balances gravity

Most importantly, its stable:

Restoring force always towards the center

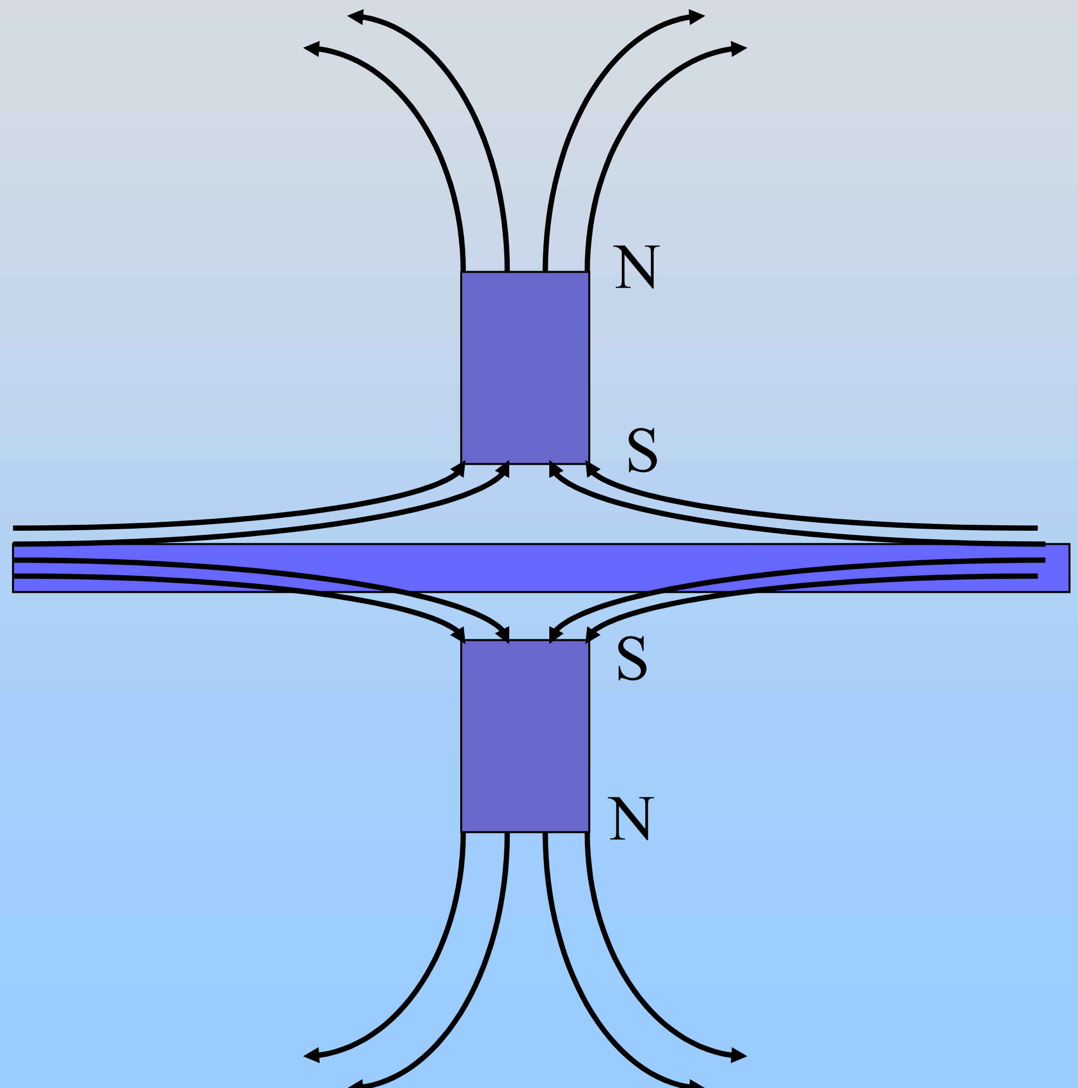
Using ∇B to Levitate

- Frog
- Strawberry
- Water Droplets
- Tomatoes
- Crickets

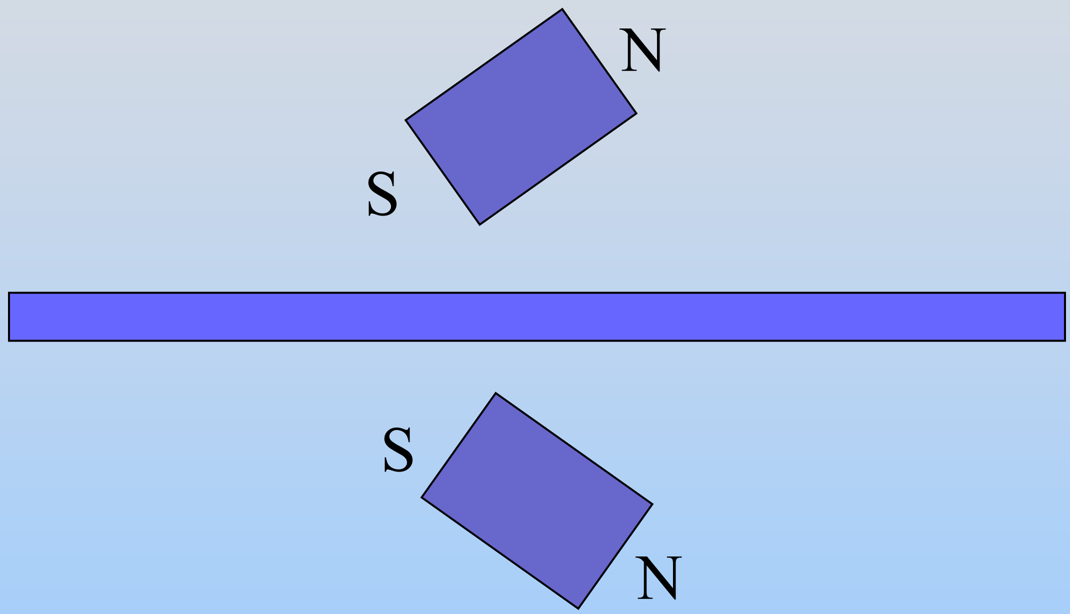
For details: http://www.hfml.ru.nl/levitation-movies.html

Demonstrating: Levitating Magnet over Superconductor

Perfect Diamagnetism: "Magnetic Mirrors"



Perfect Diamagnetism: "Magnetic Mirrors"



No matter what the angle, it floats -- STABILITY

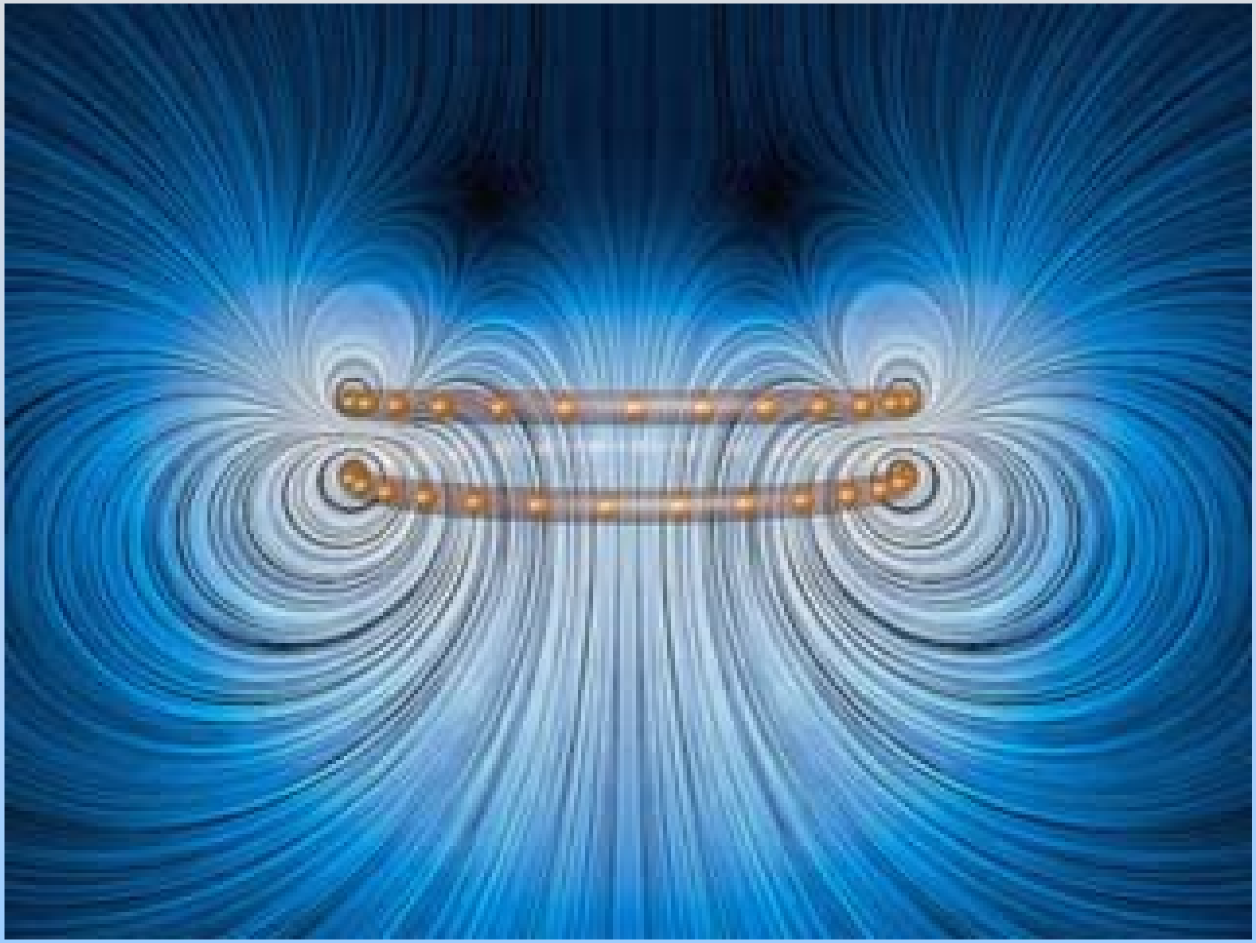
Using ∇B to Levitate

A Sumo Wrestler

For details: http://www.hfml.sci.kun.nl/levitate.html

Two PRS Questions Related to Experiment 8: Magnetic Forces

Experiment 8: Magnetic Forces (Calculating μ_0)



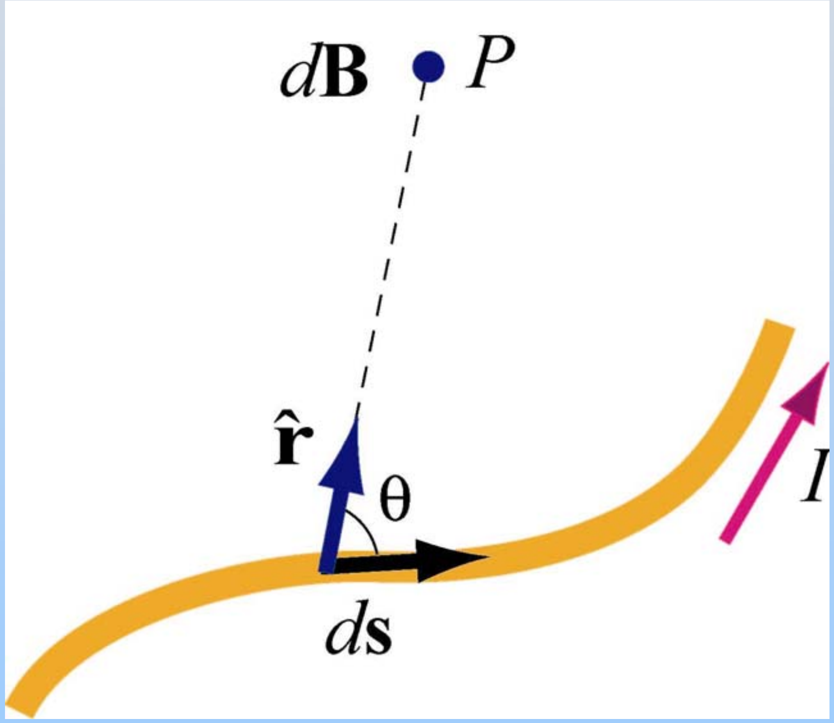
http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/16-MagneticForceRepel/16-MagForceRepel_f65_320.html

Experiment Summary:
Currents *feel* fields
Currents also *create* fields

Recall… Biot-Savart

The Biot-Savart Law

Current element of length *d**s*** carrying current *I* produces a magnetic field:

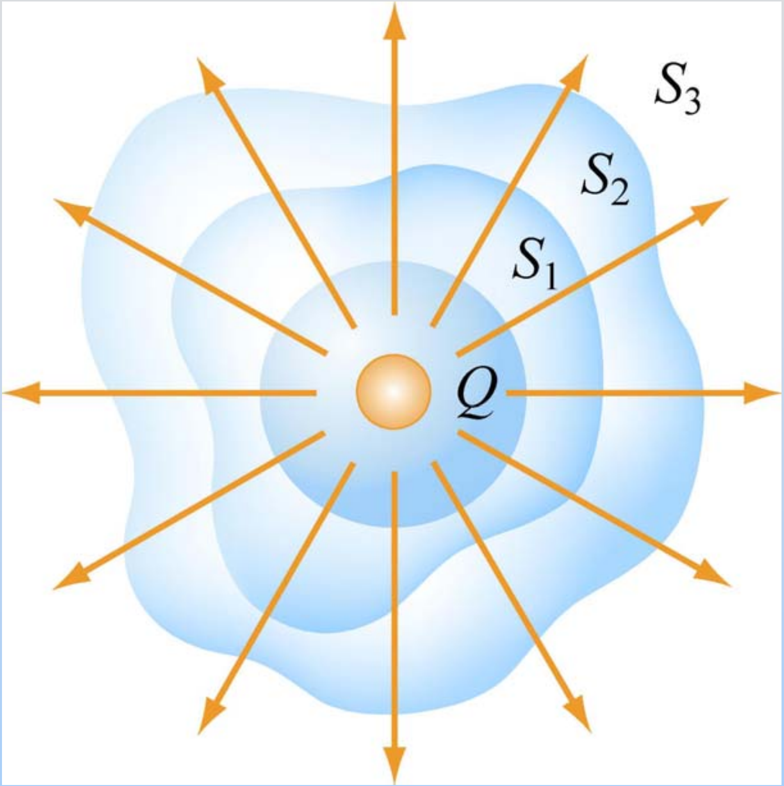


$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Today:
3rd Maxwell Equation: Ampere's Law

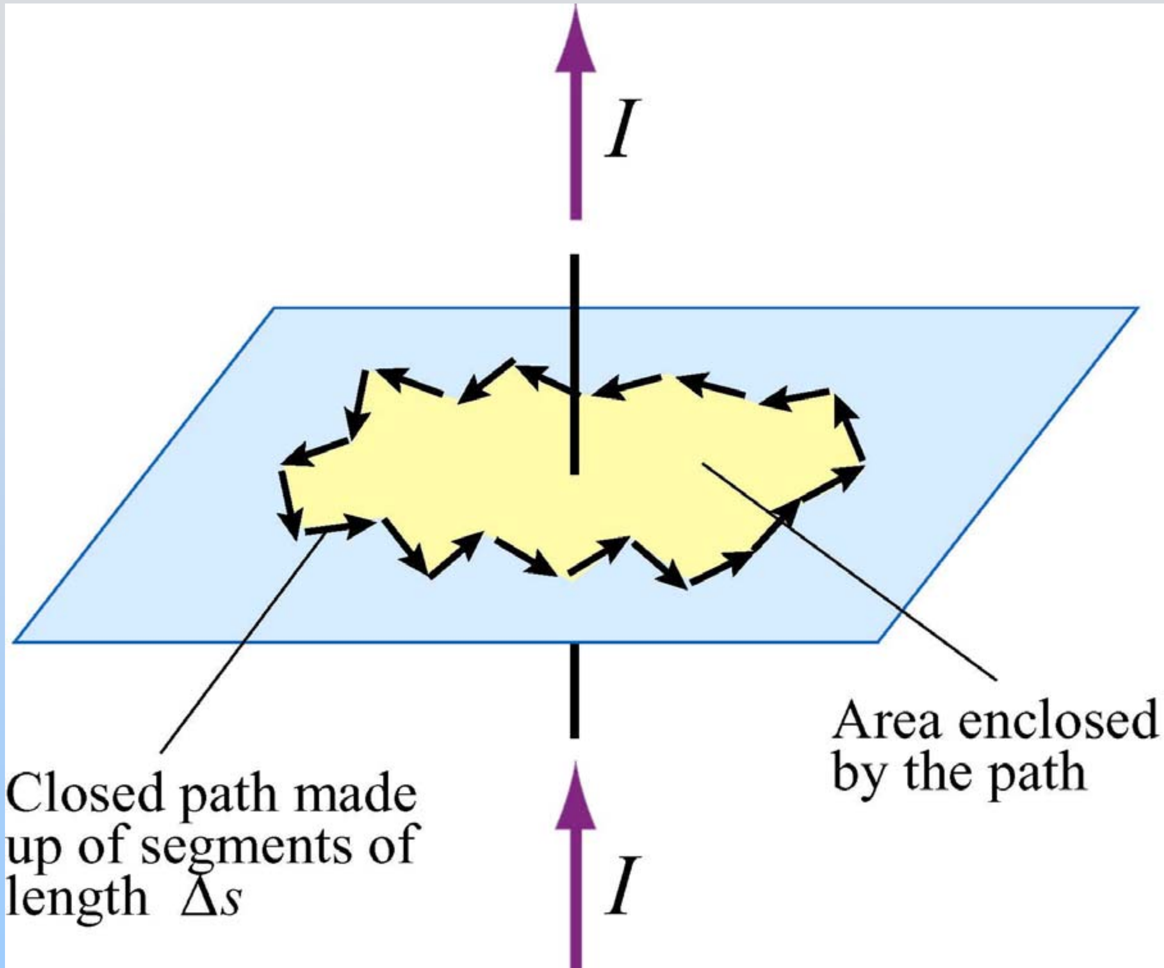
Analogous (in use) to Gauss's Law

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

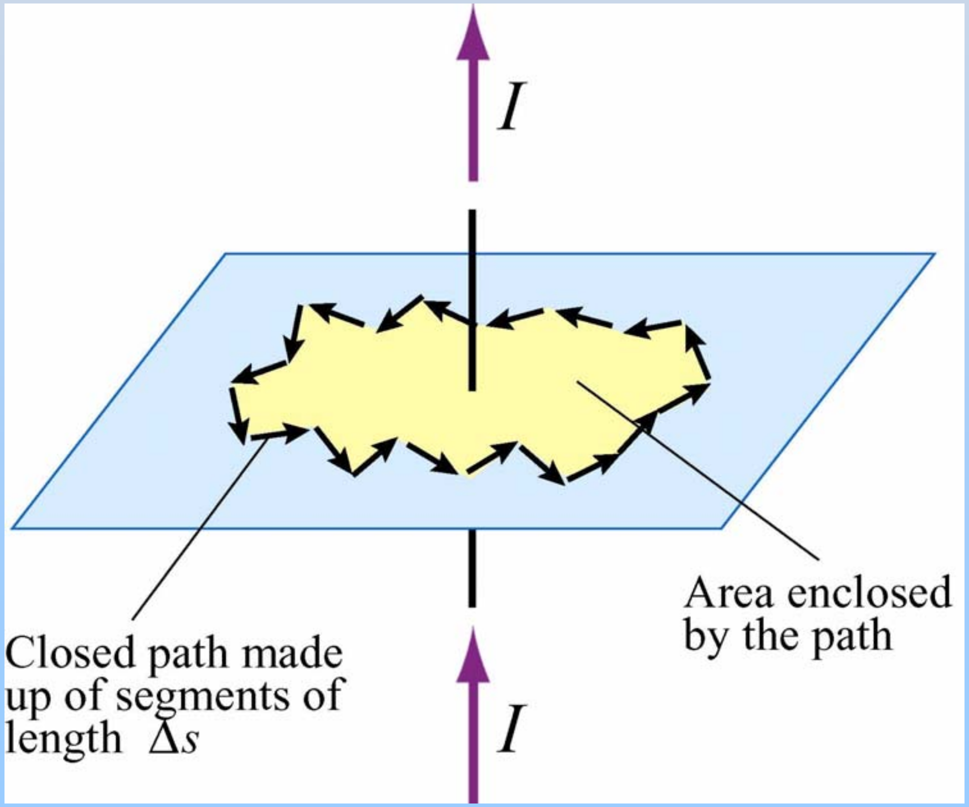
Ampere's Law: The Idea



In order to have a B field around a loop, there must be current punching through the loop

Ampere's Law: The Equation

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$



The line integral is around any closed contour bounding an open surface *S.*

I_{enc} is current through S:

$$I_{enc} = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

PRS Question: Ampere's Law

Biot-Savart vs. Ampere

Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\,\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

Applying Ampere's Law

1. Identify regions in which to calculate B field
   Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

Always True, Occasionally Useful

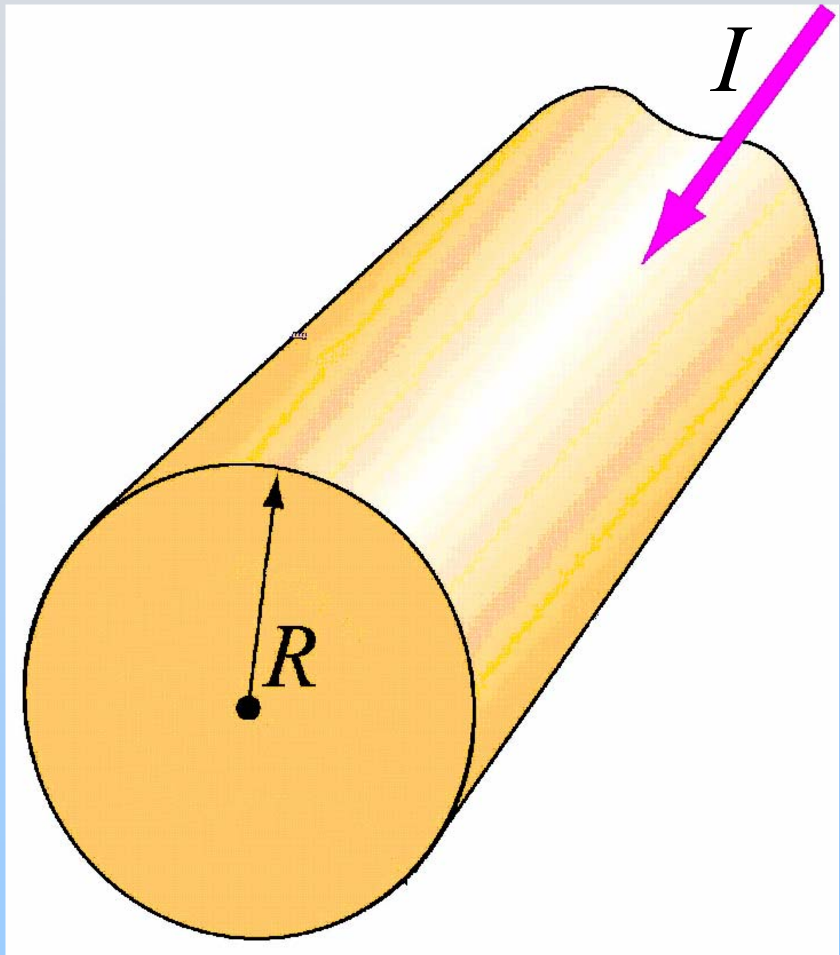
Like Gauss's Law,

Ampere's Law is **always** true

However, it is only useful for calculation in certain specific situations, involving highly symmetric currents.

Here are examples…

Example: Infinite Wire



A cylindrical conductor has radius R and a uniform current density with total current I

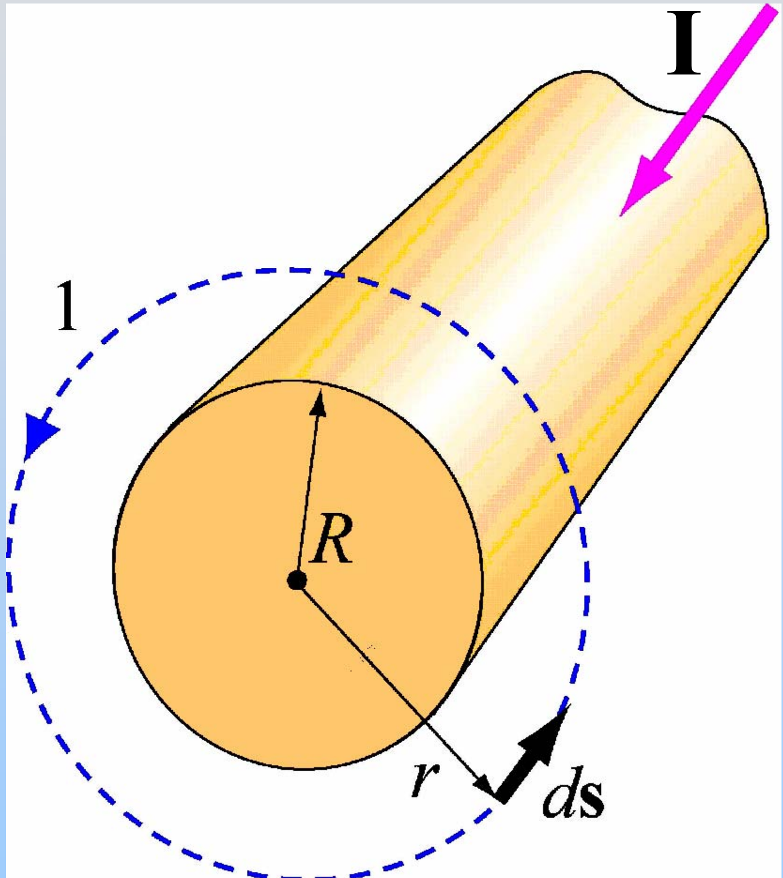
Find B everywhere

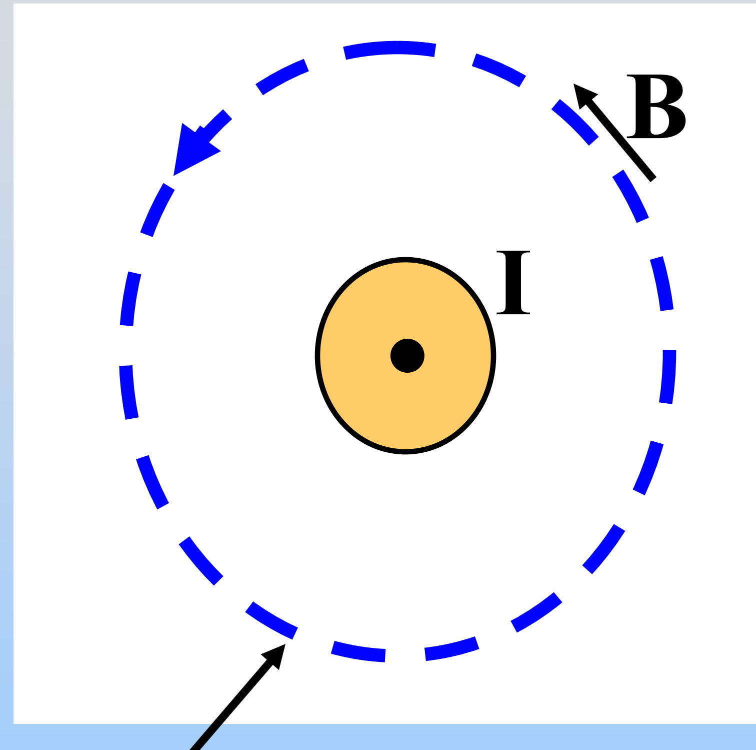
Two regions:

(1) outside wire ($r \geq R$)

(2) inside wire ($r < R$)

Ampere's Law Example: Infinite Wire



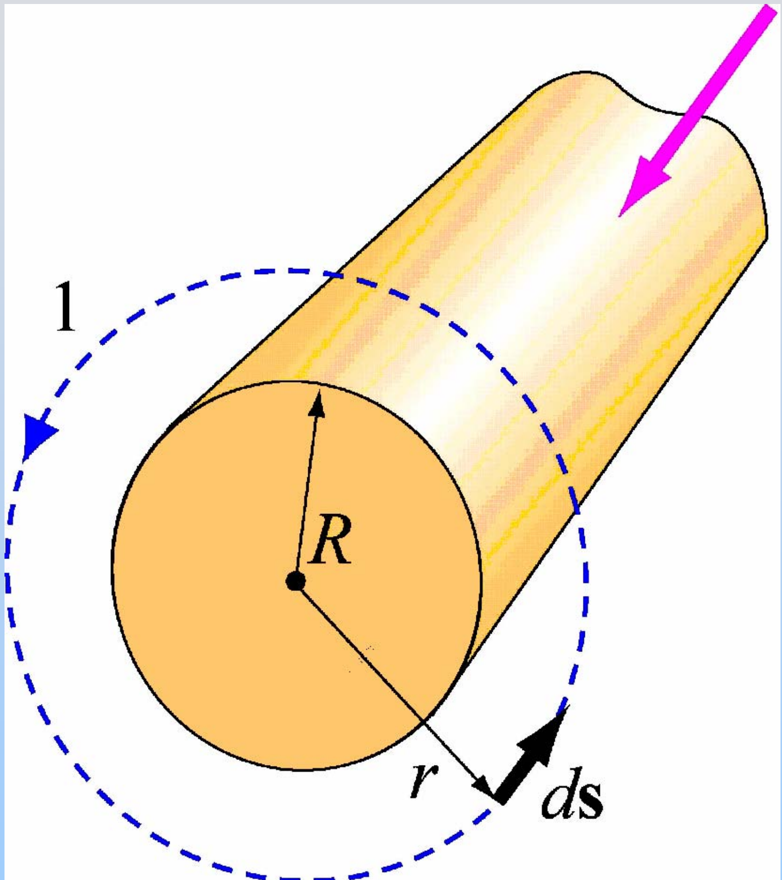


Amperian Loop:

B is Constant & Parallel

I Penetrates

Example: Wire of Radius *R*



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry →

Amperian Circle

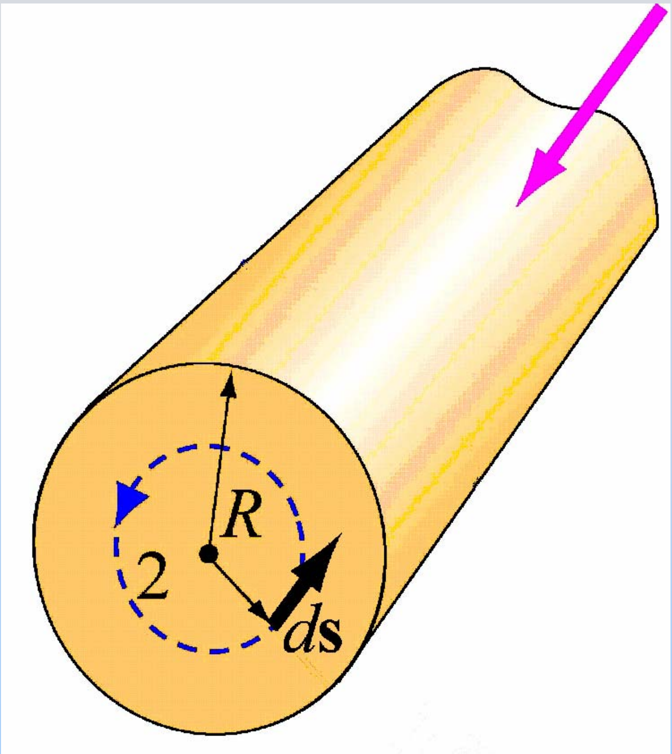
B-field counterclockwise

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B\left(2\pi r\right)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \text{counterclockwise}$$

Example: Wire of Radius *R*



Region 2: Inside wire (r < R)

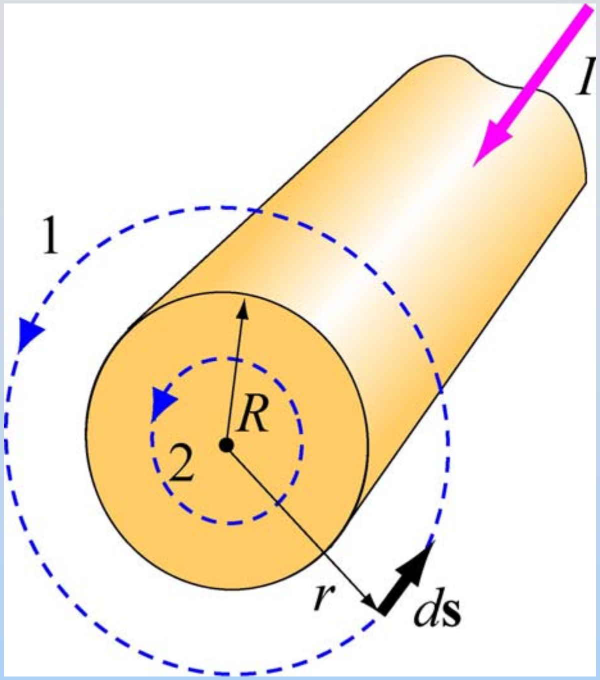
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B\left(2\pi r\right)$$

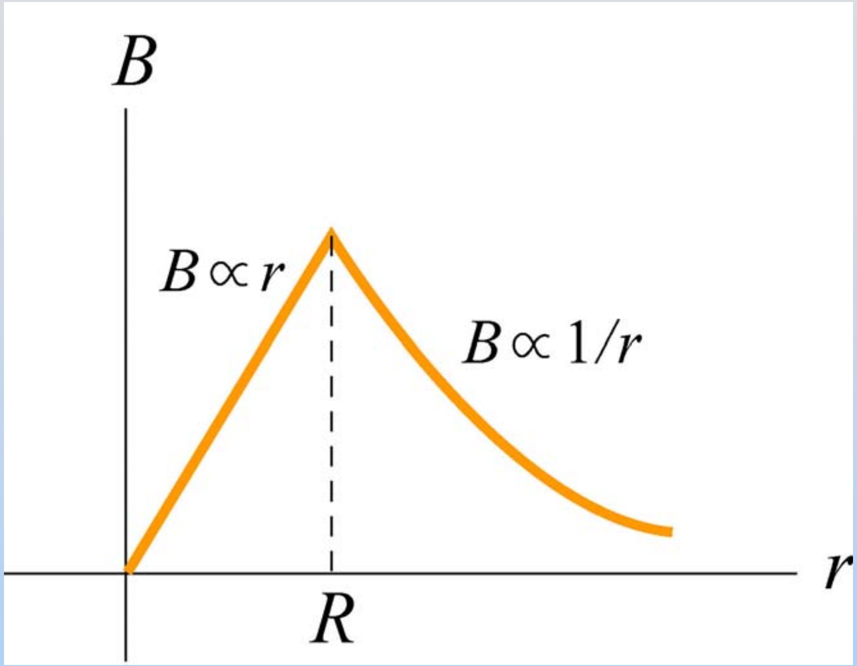
$$= \mu_0 I_{enc} = \mu_0 I \left(\frac{\pi r^2}{\pi R^2} \right)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \text{counterclockwise}$$

Could also say: $J = \frac{I}{A} = \frac{I}{\pi R^2};\ I_{enc} = JA_{enc} = \frac{I}{\pi R^2}\left(\pi r^2\right)$

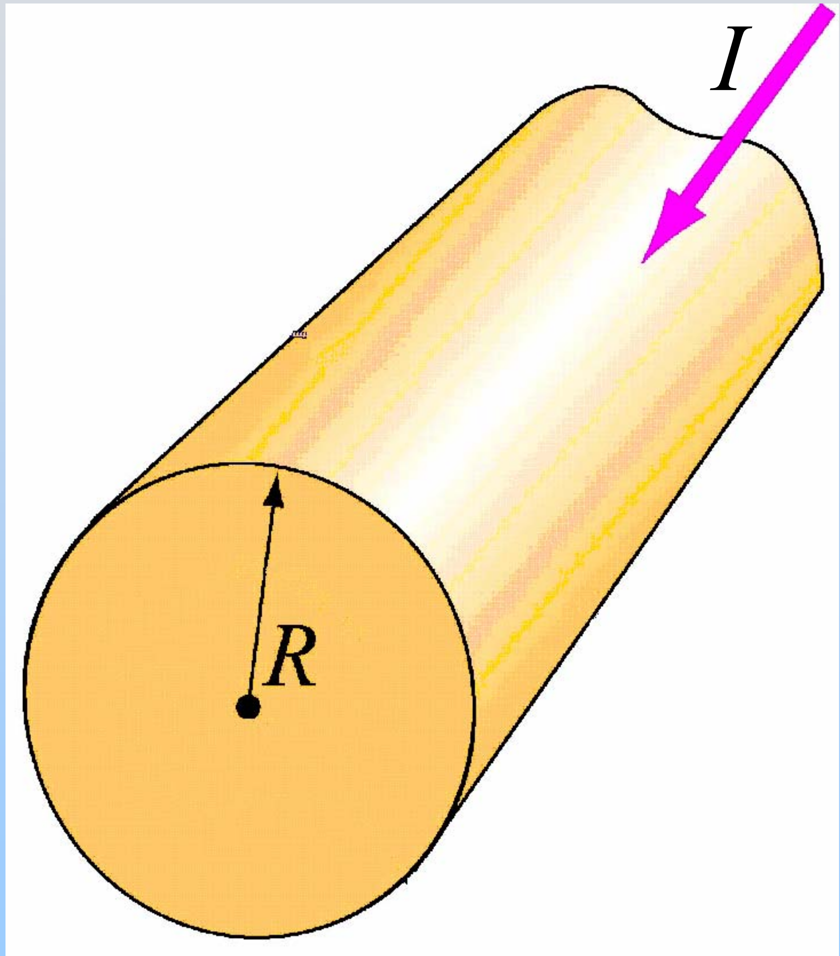
Example: Wire of Radius *R*





$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

Group Problem: Non-Uniform Cylindrical Wire



A cylindrical conductor has radius R and a non-uniform current density with total current:

$$\vec{\mathbf{J}} = J_0 \frac{R}{r}$$

Find B everywhere

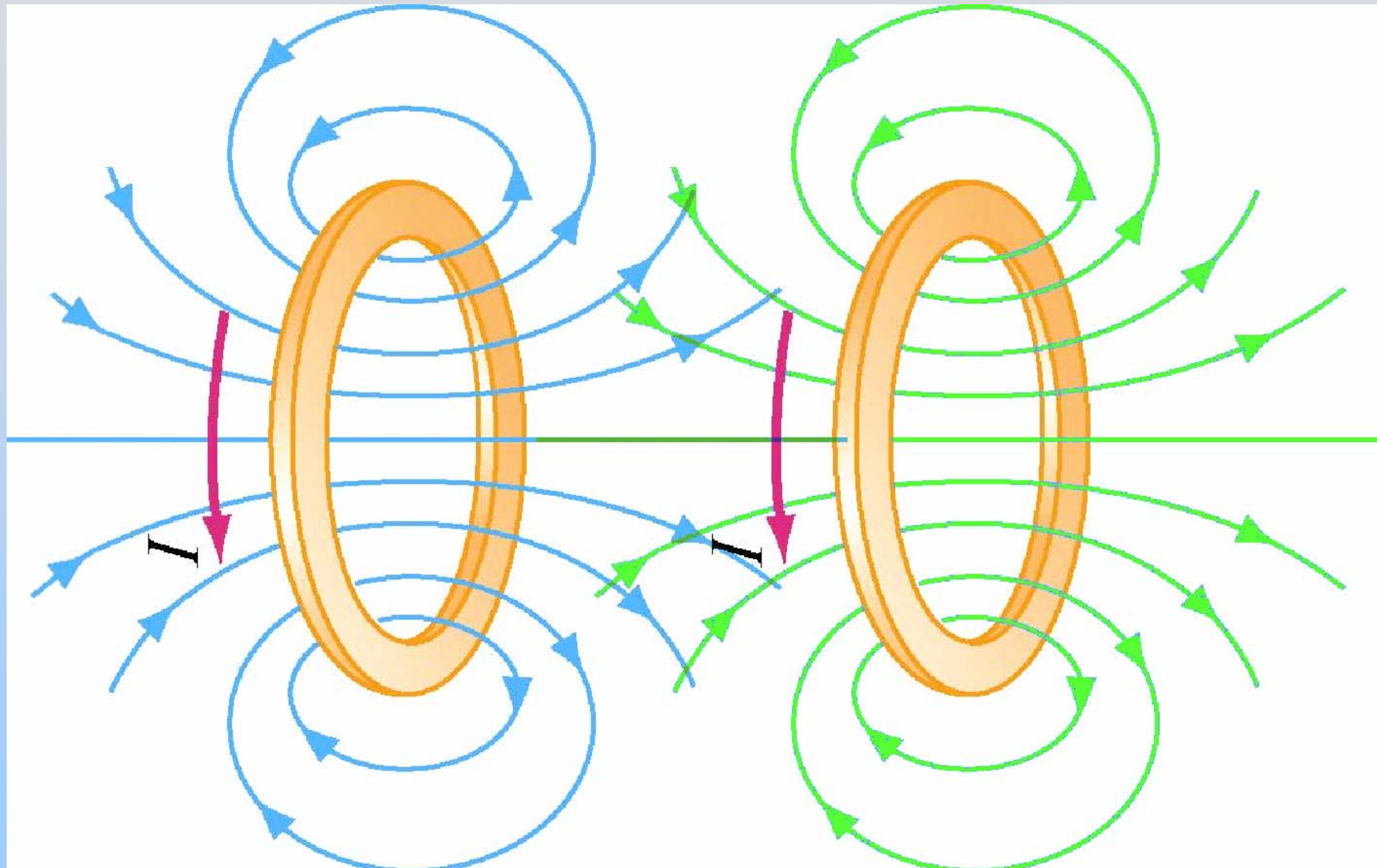
Applying Ampere's Law

In Choosing Amperian Loop:

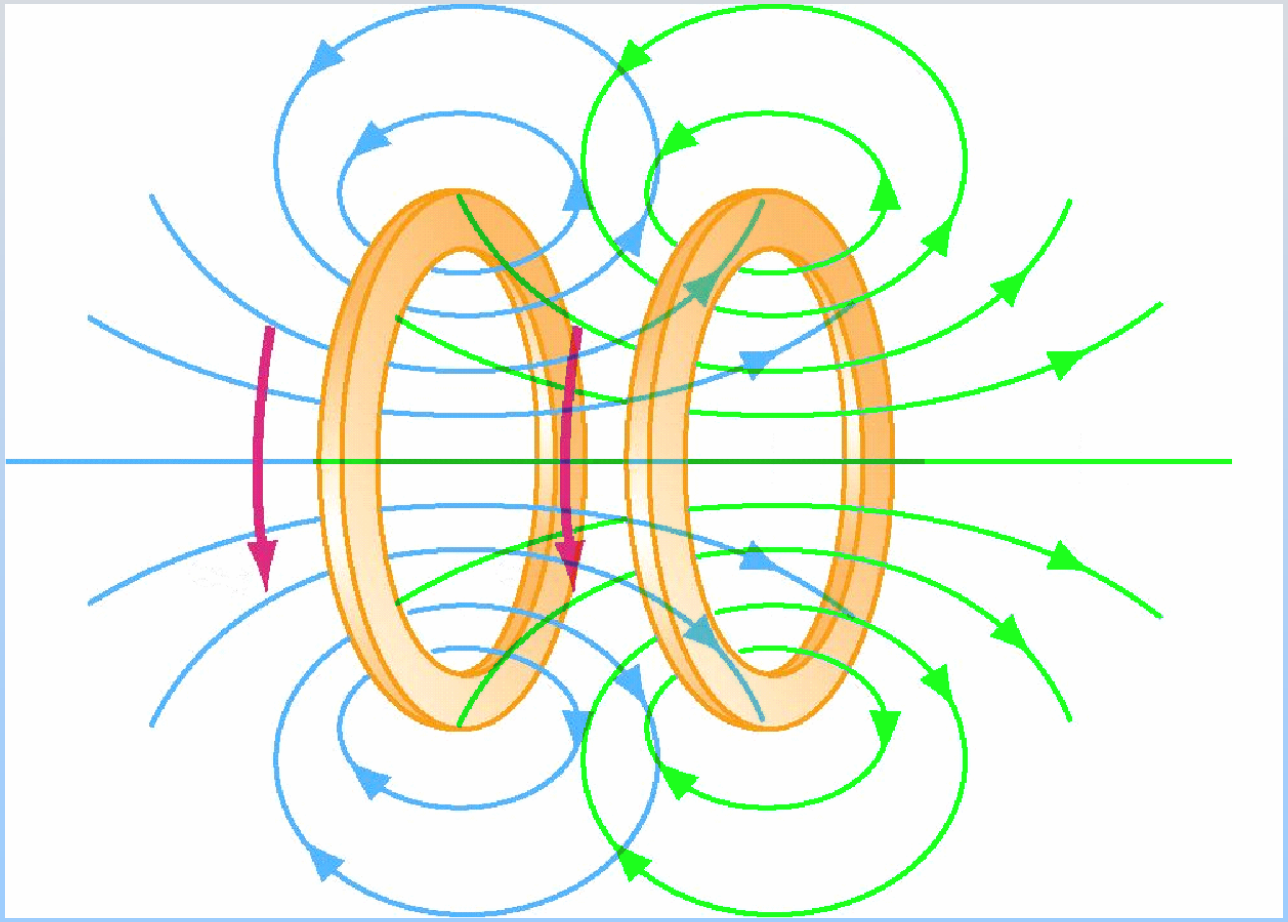
- Study & Follow Symmetry
- Determine Field Directions First
- Think About Where Field is Zero
- Loop Must
  - Be Parallel to (Constant) Desired Field
  - Be Perpendicular to Unknown Fields
  - Or Be Located in Zero Field

Other Geometries

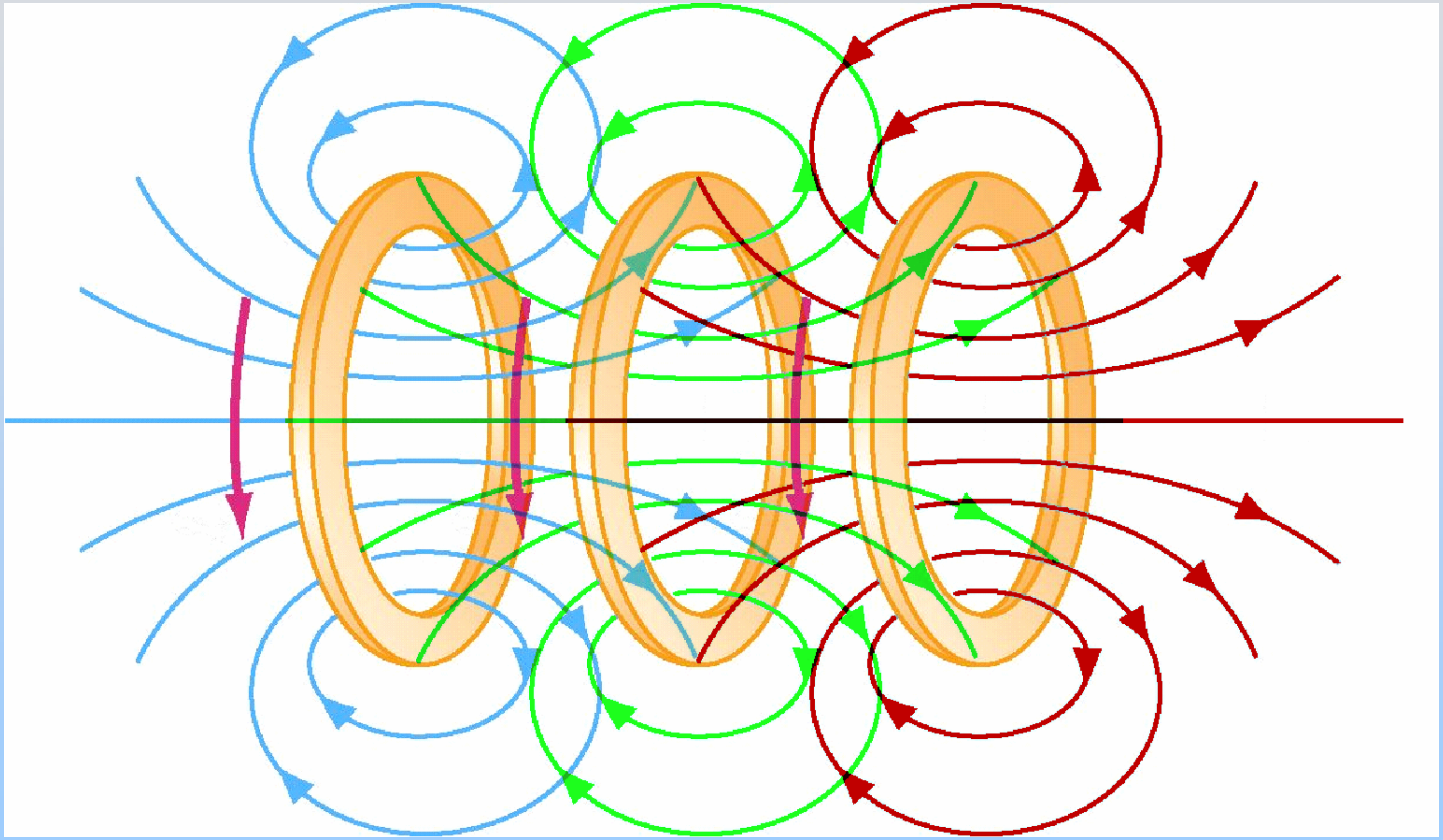
Helmholtz Coil



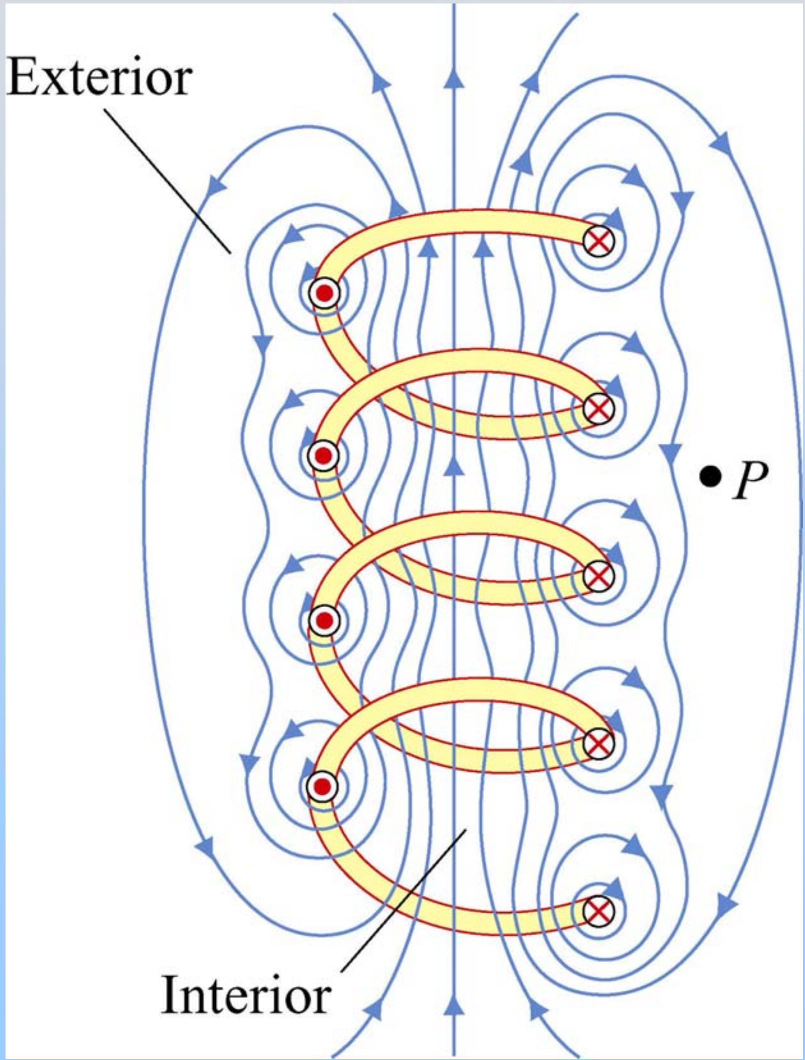
Closer than Helmholtz Coil



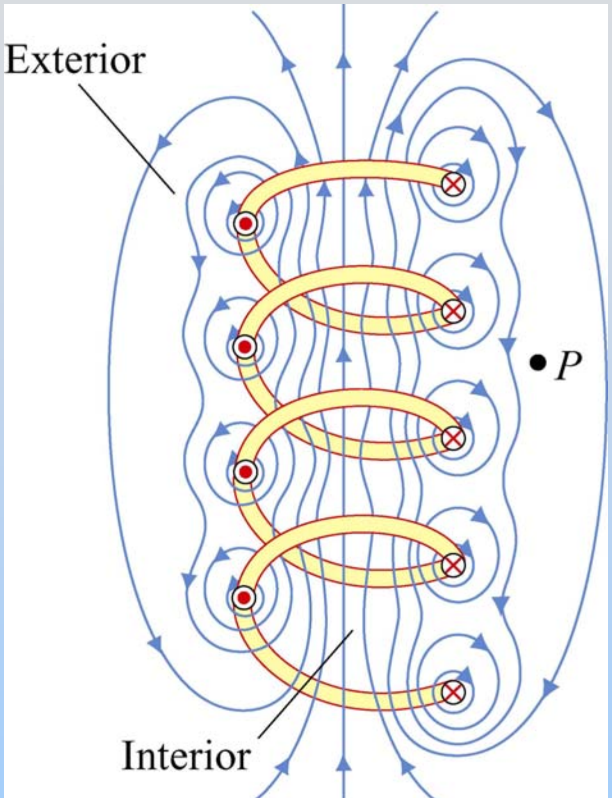
Multiple Wire Loops



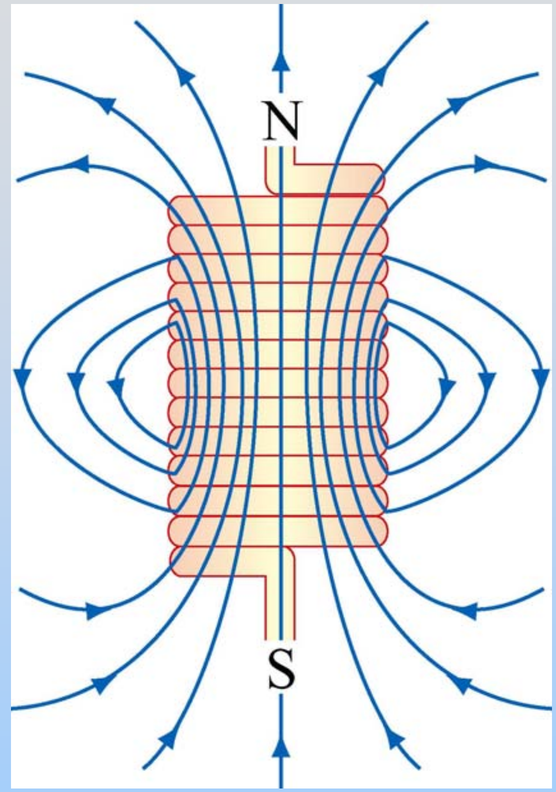
Multiple Wire Loops – Solenoid



Magnetic Field of Solenoid



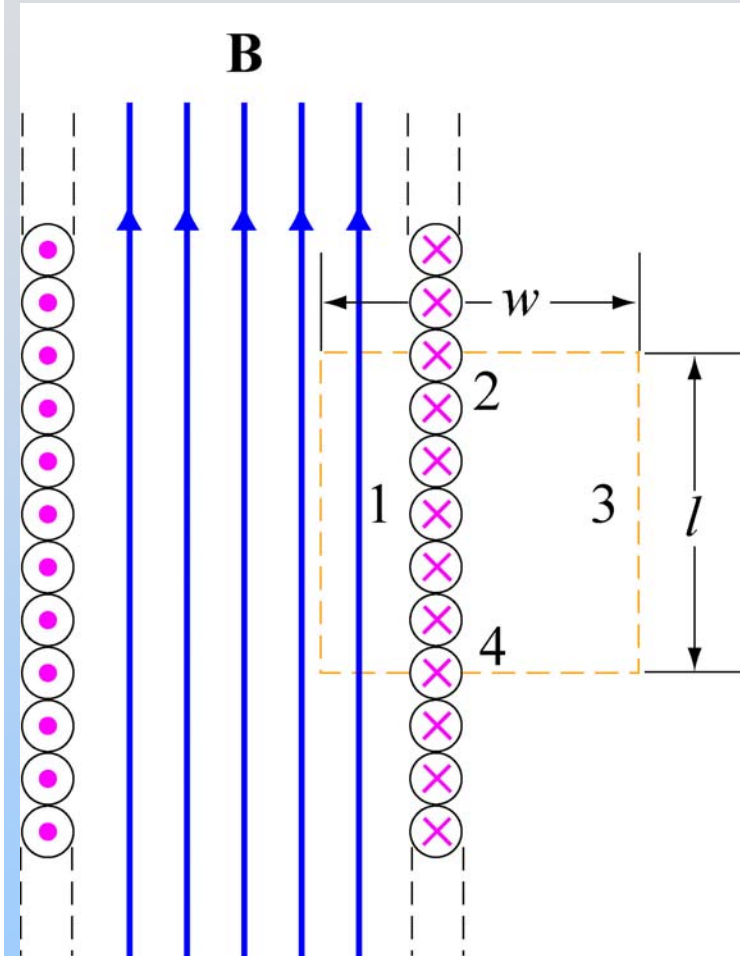
loosely wound



tightly wound

For ideal solenoid, B is uniform inside & zero outside

Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\,\vec{\mathbf{s}} \ \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \ \text{along side 3} \end{cases}$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \int_{1} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{2} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{3} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} + \int_{4} \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}}$$

$$= \quad Bl \quad + \quad 0 \quad + \quad 0 \quad + \quad 0$$

$I_{enc} = nlI$ n: turn density

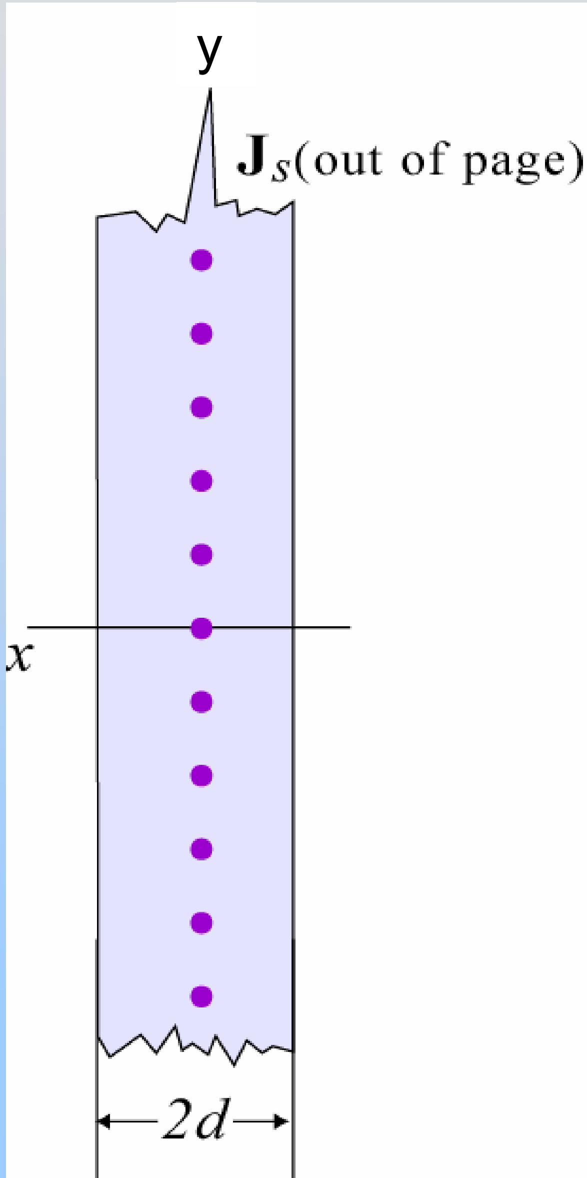
$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N / L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

Demonstration: Long Solenoid

Group Problem: Current Sheet

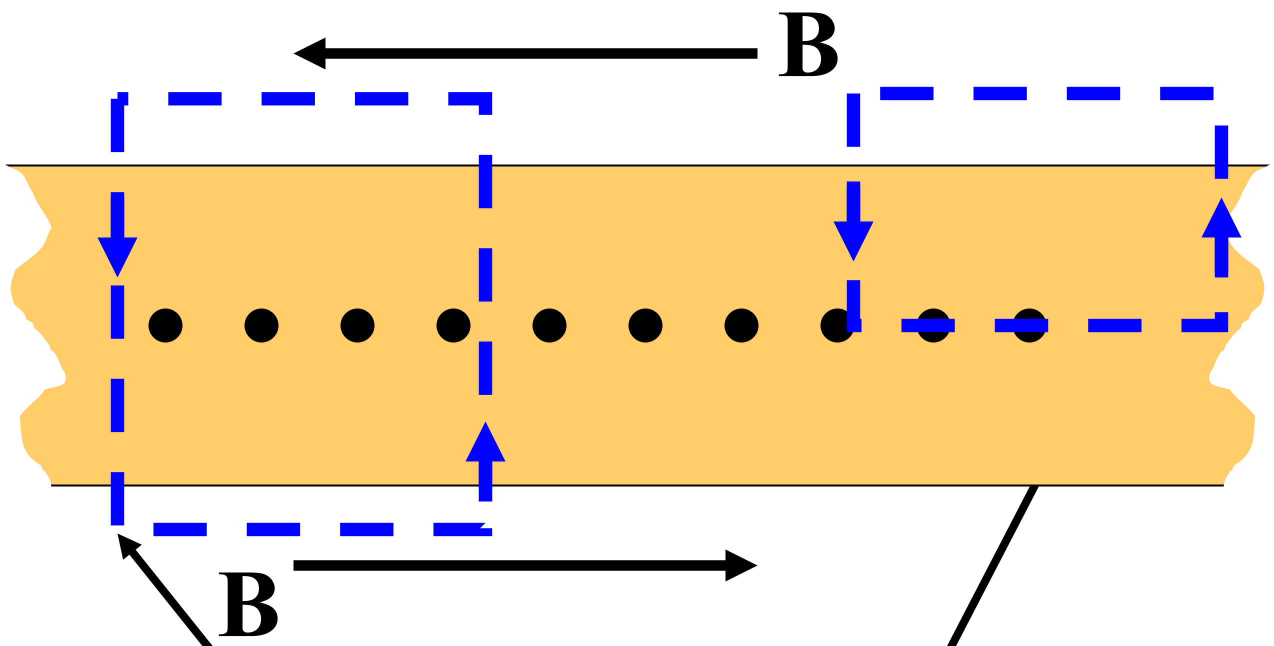


A sheet of current (infinite in the y & z directions, of thickness 2d in the x direction) carries a uniform current density:

$$\vec{\mathbf{J}}_s = J\hat{\mathbf{k}}$$

Find B everywhere

Ampere's Law: Infinite Current Sheet

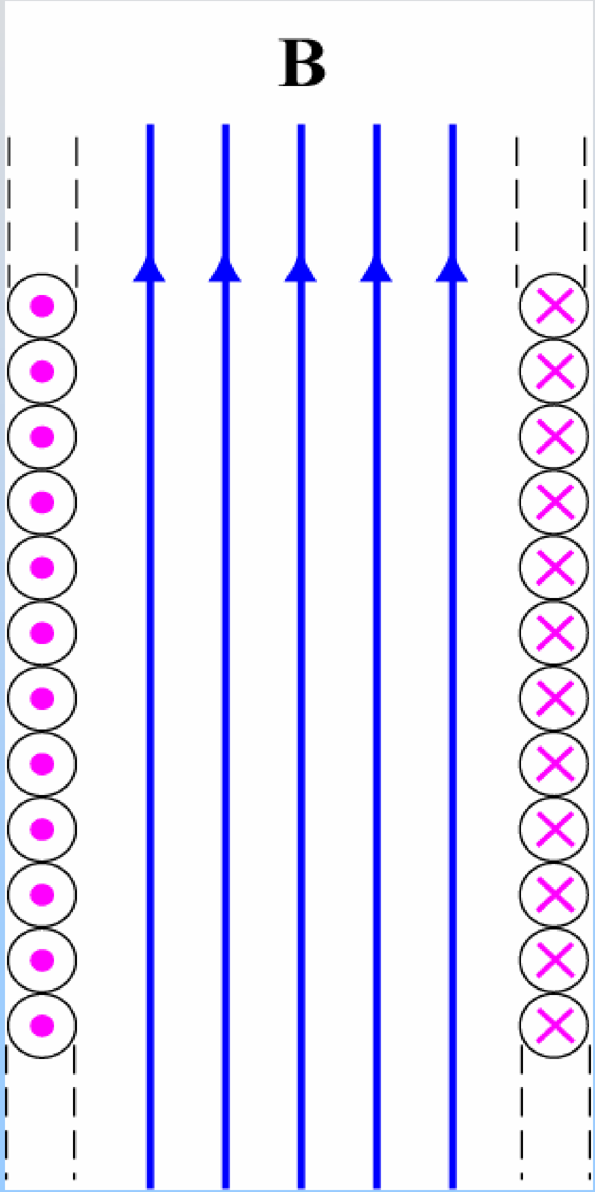


Amperian Loops:

B is Constant & Parallel OR Perpendicular OR Zero

I Penetrates

Solenoid is Two Current Sheets



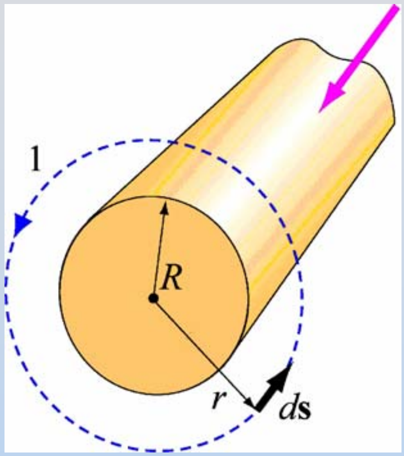
Field outside current sheet should be half of solenoid, with the substitution:

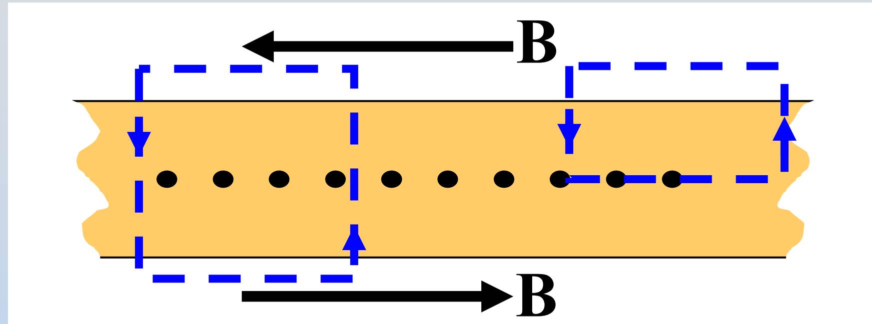
$$nI = 2dJ$$

This is current per unit length (equivalent of λ, but we don't have a symbol for it)

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

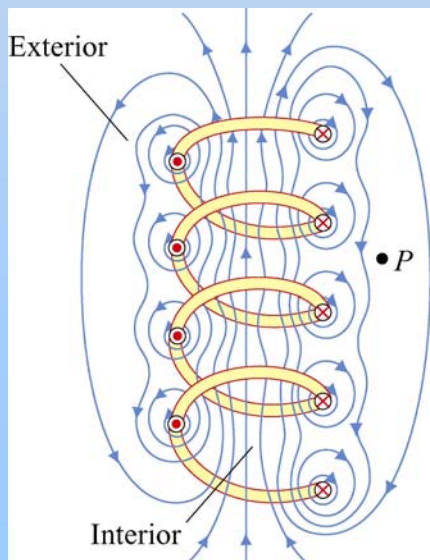
Long Circular Symmetry

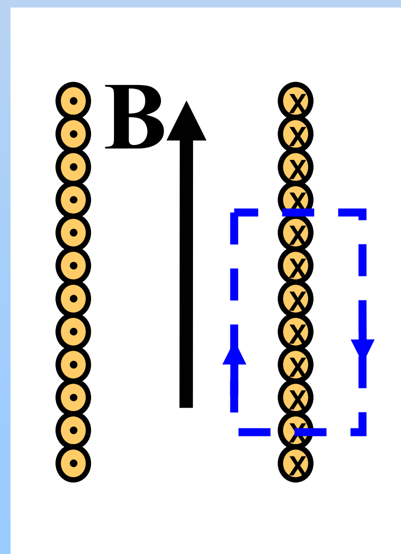


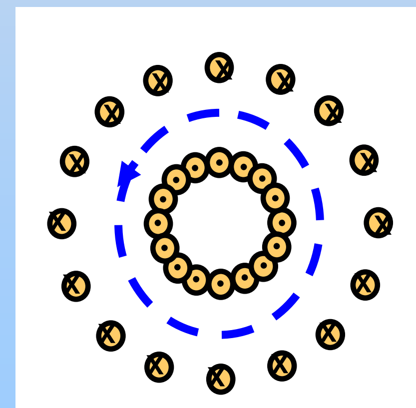


(Infinite) Current Sheet

Solenoid = 2 Current Sheets







Torus

Brief Review Thus Far…

Maxwell's Equations (So Far)

Gauss's Law: $\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: $\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

No Magnetic Monopoles! (No diverging B Fields)

Ampere's Law: $\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc}$

Currents make curling Magnetic Fields