Experiment EF—Electrostatic Force

Introduction

In this experiment you'll investigate aspects of the electrostatic force. This force has such varied roles as making currents flow in wires, holding atoms and molecules together, making clothes cling, glue stick, etc. The strength of the electrostatic force is proportional to the product of a charge (measured in coulombs) and the electric field (measured in volts/meter) produced by the other charges.

You'll set up a horizontal parallel plate capacitor and find the voltage at which a piece of aluminum foil of known weight just lifts off the bottom plate. The electric force then just balances the force of gravity,

$$\vec{F}_{electric} = \vec{F}_{gravity} \, .$$

Besides a single thickness of foil, you will use folded foils equivalent to n thickness as well. The electric field between the plates is proportional to the surface charge density on the plates. When the electric field is measured in SI units, the constant of proportionality between the field and the surface charge density is determined by ε_0, the permittivity of free space. From the relations between measured or known electrical quantities, material properties and apparatus dimensions you can determine the constant ε_0.

Theory

You want to equate the gravitational and electric forces on the foil. The gravitational force is just the weight of the foil. The density ρ of the aluminum foil material is 2.7 grams/cc ($2.7 \times 10^3 \, kg/m^3$). The volume of the foil is then its area, A_{foil}, times its thickness t. Therefore the magnitude of the gravitational force on the foil is

$$F_{gravity} = m_{total} \, g = \rho t A_{foil} \ g \, .$$

Finding the electrical force is a bit subtler. Treat the closely spaced, metal portions of the washers as a parallel-plate capacitor. The electric force on the foil will be its charge times the electric field it feels. The subtlety lies in calculating that field: The total field in the capacitor is just V/d where V is the applied voltage and d is the spacing between the plates. But only half that field is due to charges on the upper plate; and the other half is due to charges on the lower plate.

Gauss's Law, states that the electric flux through a Gaussian surface is equal to the charge enclosed in a Gaussian surface divided by the constant ε_0.

$$\oiint_{\substack{Gaussian \\ surface}} \vec{E} \cdot d\vec{a} = q_{enclosed}/\varepsilon_0$$

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Choose a cylindrical Gaussian surface whose upper end is inside the top plate and whose bottom end has area a and is within the gap. (See Figure 1.)
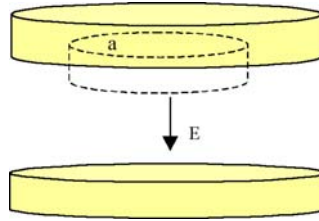

Figure 1: Gaussian surface for parallel discs

The electric flux through the Gaussian surface is just the constant electric field inside the gap, E, times the area, a, of the end cap. (Note the electric field is zero on the end cap that is outside the surface). The charge enclosed by the Gaussian surface is a times the charge density σ (*charge/area)* on the upper plate. So applying Gauss's Law yields that the total electric field in the gap is

$$E = \sigma / \varepsilon_0 .$$

Therefore the charge density, σ, can be found by equating the total field, $E = V/d$, to the field found from Gauss's Law:

$$\sigma = \varepsilon_o \frac{V}{d} .$$

To find the electric force on the foil, assume that the charge density, σ, on the foil, is the same as that of the lower washer. Here comes the subtle point. By thinking about the electric field from the charge sheets on the inner surfaces of the plates (positive on top and negative on the bottom) you come to the conclusion that 1/2 the total field comes from the upper plate and half from the lower plate. Charges on the foil feel only horizontal forces from other charges on the bottom plate, so the vertical force on the foil is due to the electric field of just the top charge sheet. The magnitude is given by:

$$F_{electric} = Q_{foil} E_{top} = \sigma A_{foil} \frac{V}{2d} = \frac{\varepsilon_o V^2 A_{foil}}{2 d^2}$$

Equating the electric and gravitation forces yields,

$$\frac{\varepsilon_o V^2 A_{foil}}{2d^2} = \rho t A_{foil} \, g .$$

The area of the foil, A_{foil}, cancels giving:

$$V^2 = \left(\frac{2d^2 \rho g}{\varepsilon_0}\right) t \quad .$$

If you plot V^2 vs. t, you should get a straight line whose slope is the coefficient of t. You can calculate the free permittivity of space, ε_0, from your experimental value for the slope:

$$\varepsilon_0 = \frac{2\rho g d^2}{slope} \quad .$$

Use the following values:

- thickness of perf-board + tape, $d = 1.7 \times 10^{-3} m$;
- thickness of Aluminum foil, $t = 7.6 \times 10^{-6} m$;
- density of Aluminum foil, $\rho_{Al} = 2.7 \times 10^3 kg/m^3$;
- acceleration due to gravity, $g = 9.8 m/s^2$.

Apparatus

First make a parallel plate capacitor from two large washers (2.5 inch diameter). The experiment depends on the smoothness and cleanliness of the washers and the cleanliness of the foil. You can't do much about the smoothness of the washers beyond choosing, by sight and touch, the best areas for where you put the foil (use a finger nail to feel the surface of the washer). Rub the washers vigorously with a paper towel to remove "finger grease" or other crud. Break the piece of perf-board into three equal sized pieces. Tape the 3 pieces of perf-board to one of the large washers at about 120° separations to form spacers that establish the gap between the washers. (Use black insulating tape.)

Figure 2: perf-board used as spacers on washer

The perf-board is $1.5 mm$ thick, and the tape is $0.2 mm$ thick. Since you want the gap to be as uniform as possible, avoid burrs on the perf-board and smooth the tape down carefully. Press the second washer down on the spacers and check the gap measurement as best you can

with your steel scale to be sure it's not grossly off. Tape two #30 wires to the outside of the washers, and place the first washer, the one with spacers, level on your desk.

Use one of your MMM connected across the output of the HVPS to read the HVPS output (the right MMM in Figure 3). Connect the lower washer to the low voltage side of the HVPS. Connect the upper washer to the negative input of the second MMM. Connect the positive input of the second MMM to the high voltage side of the HVPS. This puts the second MMM in series with the gap (the left MMM in Figure 3). You can use electrical tape to hold these connections firmly on the washers. Both MMM are set on the 1000DCV range.
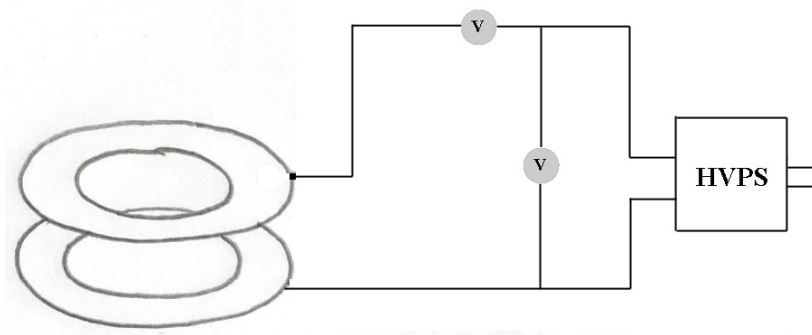

Figure 3: Experiment EF connections

Experiment

Cut a square piece of foil, about $1cm \times 1cm$. The foil should also be clean and flat, but it also needs to be given a rough texture to allow air to pass under it when it lifts off. Press it firmly on the 80 grit sandpaper found in the Red Box with a piece of tissue between foil and finger. Try to handle the foil with your fingers as little as possible to keep it clean. Tweezers work well for handling the foil. Alternately, you can tip it off the abrasive cloth onto a piece of paper and then slide it onto the washer. Transfer the foil to the bottom washer, and make sure that it's fairly flat. You can see that if the foil isn't flat, it either changes the gap or charge will flow out to the part nearest the upper washer so that larger forces will be exerted than if the foil were flat.

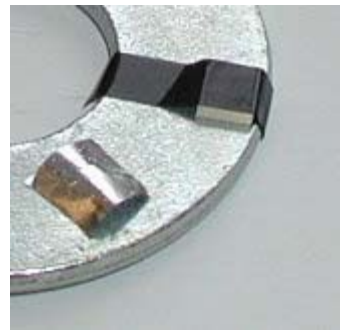

Figure 4: Charge flows to edge of foil

Figure 5: Charge flows to center of foil

Put the top washer in place and turn up the HVPS all the way. The foil should lift off and bounce around. Turn down the voltage and repeat. When the foil lifts and shorts the gap, the

second MMM will register a voltage. Once the apparatus is working well, carefully and slowly raise the voltage and read the voltmeter when the foil lifts off. Read the voltage across the gap by the first MMM which is connected directly across the HVPS output. There are two other ways to determine when the foil lifts off the plate.

- Directly, by looking in the gap, seeing the foil lift, and having your partner read the voltmeter at that instant.

- Watch the MMM readings as you slowly increase the voltage. When the foil lifts and shorts the gap, the MMM reading will drop.

Experiment EF Parts

1 strip   perfboard
2   washers, 1in. plated steel
4 ft wire, #30 bare solid
1piece   Al foil, $2in$ square $0.0003in$ thick
1piece   sand paper 80 grit

Figure 6: Experiment EF parts

Experiment EF Data Analysis

1. Once the apparatus is working well, carefully and slowly raise the voltage and read the multimeter when the foil lifts off. Gentle tapping may help to get more reproducible results. Repeat 5 times. Take care that the foil isn't shorting the gap when you read the meter. (If you are having trouble, try different sizes of foil.) Prepare double and triple thickness foils (again about $1cm \times 1cm$ from pieces $2cm \times 1cm$ and $3cm \times 1cm$, folded and squeezed flat, and repeat the above procedure.

2. *Analysis*: Average the voltages for a given foil thickness. You then have 3 numbers: the voltages required to lift 1, 2, and 3 thicknesses of foil. **Use a table like this to enter your Experiment EF data**

Number of foils	Voltage Trial 1	Voltage Trial 2	Voltage Trial 3	Voltage Trial 4	Voltage Trial 5	Average Voltage
1						
2						
3						

3. Plot the voltage <u>squared</u> V^2 vs. n, the number of thicknesses of foil. Is the origin a data point? (Zero voltage will lift zero folds of foil). **Include your graph**.

4. Your graph of the experimental values of V^2 vs. n should be a straight line with the slope being the factor in parentheses. **Find the slope of the best-fit line.**

5. Calculate the free permittivity of space ε_0, from your experimental value for the slope using the result that

$$V^2 = \frac{\rho tg 2d^2}{\varepsilon_0} n .$$

Problem 1: *Electrostatic Force Experiment*

a) Using Gauss' Law, find an expression for the electric field between two discs of radius R that are separated by a distance d. The discs have opposite charges that are equal in magnitude placed on them. You may neglect edge effects. Make a sketch of the electric filed lines when you include edge effects.

b) Suppose a voltage difference ΔV is applied across the two discs. Show that the charge on the positive plate is given by the expression, $Q = \dfrac{\varepsilon_o \pi R^2 \Delta V}{d}$ when you ignore edge effects.

c) Taking into consideration edge effects, the actual expression for the charge is given by $Q = f \dfrac{\varepsilon_o \pi R^2 \Delta V}{d}$ where f is a correction factor that depends on the ration of separation to plate radius. For circular plates, the factor f depends on d/R as follows:

d/R	f
0.2	1.286
0.1	1.167
0.05	1.094
0.02	1.042
0.01	1.023

Explain why the factor f is always greater than 1. Where is the 'extra charge'?

d) Find an expression for the capacitance of the circular discs neglecting edge effects. Now take into consideration the correction factor f. Is the actual capacitance larger or smaller?

e) How would you expect your results in parts a) – d) to change if you used two washers of inner radius r_1 and outer radius r_2 instead of two discs?

Problem 2: *Internal Resistance of the HVPS*

In experiment EF, you connect the lower washer to the minus side of the HVPS, and the upper washer to the plus side. Let's approximate the internal resistance of the HVPS by $0.2M\Omega$ and the two $1M\Omega$ gives a total internal resistance $R_T = 0 = 2.2\,M\Omega$. Use one multimeter on the $1000V$ DC setting to read the HVPS output. Remember that this also acts as a $20M\Omega$ resistor. Connect the second multimeter on the $1000V$ DC setting in series with the gap. Again this acts as a $20M\Omega$ resistor. When the foil lifts and shorts the gap, the second multimeter does two things: it will register a voltage, and it acts as a current limiter due to its high resistance. The circuit is shown in figure 1 below.
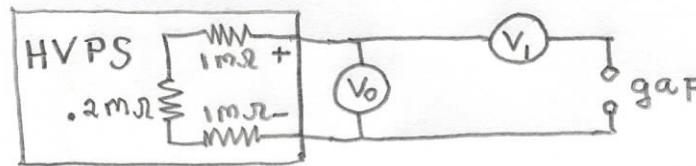

Figure 7: Experiment EF circuit

The voltage across the first multimeter will drop due to the changed load as soon as the foil lifts. Just before the foil lifts, suppose the voltage is $V_{HVPS} = 400V$.

a) What is the current through the multimeter when it is connected across the output of the HVPS?

b) Find a relation between the emf of the high voltage power supply, V_{emf}, the current in the circuit, I, the output voltage across the HVPS, V_{HVPS}, and the total internal resistance of the HVPS, R_T.

c) When the foil lifts, what is the equivalent load resistance due to the two multi-meters? Are they in series or parallel?

d) What is the total resistance of the circuit including the total internal resistance of the HVPS?

e) What is the current that flows in the circuit?

f) Once the foil has lifted, what is the new voltage across the terminals of the HVPS?