Massachusetts Institute of Technology
Physics 8.03SC Fall 2016
Homework 4

Problems

Problem 4.1 (25 pts)

In the beaded string shown in Figure 1, the interval between neighboring beads is a, and the distance from the end beads to the wall is $a/2$. All the beads have mass m and are constrained to move only vertically in the plane of the paper. The strings are massless with constant string tension T.

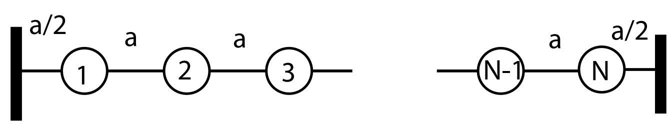

Figure 1: Beads on a string

a. Show that the physics of the left-hand wall can be incorporated by going to an infinite system and requiring the boundary condition $A_1 = -A_0$.
b. Find the analogous boundary condition for the right-hand wall.
c. Find the normal modes and the corresponding frequencies for the finite system.

Problem 4.2 (25 pts)

A physicist was trying to understand a system with two coupled torsional pendula. First, she tried to write down the equation of motions of the pendula in terms of θ_1 and θ_2, which are the angles with respect to the equilibrium positions of the pendula. She found that the interaction matrix $\mathcal{M}^{-1}\mathcal{K}$ commutes with the 2×2 reflection symmetry matrix

$$\mathcal{S} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i.e., satisfy this condition $[\mathcal{S}, \mathcal{M}^{-1}\mathcal{K}] = 0$)

a. How many normal modes do we have in this system?
b. What will be the amplitude ratio (A_1/A_2, where A_1 and A_2 are the components of the eigenvector) of the torsional pendula in each normal mode?

Problem 4.3 (25 pts)

Consider a uniform thin string of length L and mass density μ. The string is attached at both ends to vertical, frictionless rods via massless rings as shown in Figure 2. The tension in the string is T.

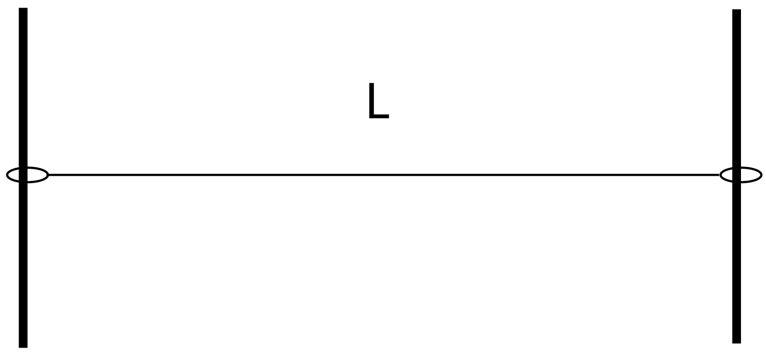

Figure 2: Uniform String

a. Find the normal modes and their frequencies for small amplitude transverse oscillations.
b. Sketch the shapes of the first three normal modes.
c. Make a graph of the angular frequency ω as a function of the angular wave vector k.

Problem 4.4 (25 pts)

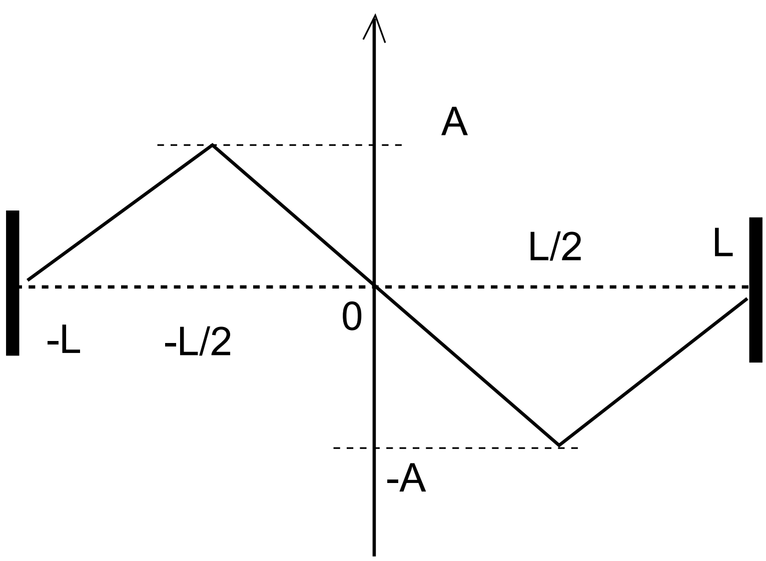

Figure 3: String Deformation

A string of mass density μ and stretched with tension T is deformed as shown in the Figure 3 (amplitude A is very small, the deformation as shown in the figure is greatly exaggerated). The string is released at $t = 0$ with zero initial velocity ($\frac{\partial y(x,t)}{\partial t} = 0$ at $t = 0$ for all x).

a. How many normal modes do we have in this system?

b. Find expressions for the Fourier coefficients A_m of the normal mode expansion of the initial string shape and use them to write a full time-depenent series that describes motion of the string at $t > 0$: $y(x, t)$.

c. Make a sketch of the amplitudes A_m of the modes as a function of mode number m. Make sure that you indicate amplitudes for all possible normal modes of the string.