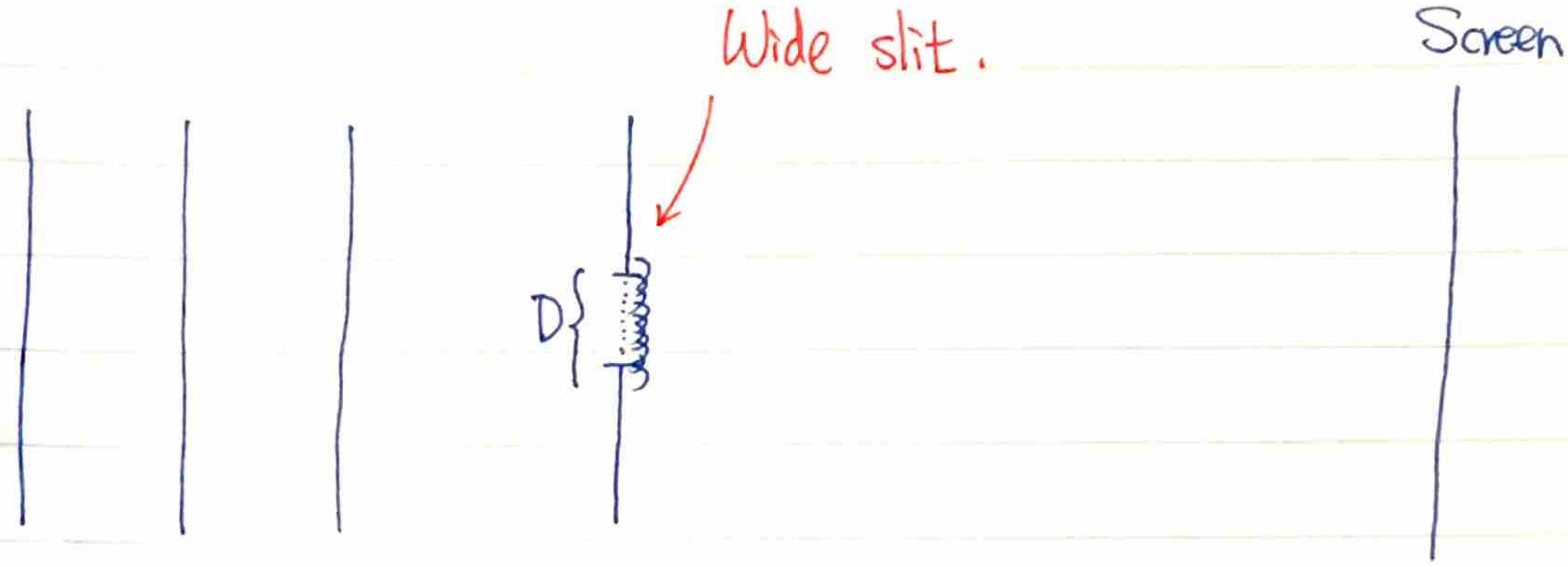
We learned the interference of 2 EM waves
N EM waves

$\boxed{EM} \rightarrow \boxed{2EM} \rightarrow \boxed{N\text{-}EM} \rightarrow \boxed{\infty\text{-}EM}$

Interference of infinite number of EM waves.

"Diffraction"



We have ∞ point like sperical EM wave sources.

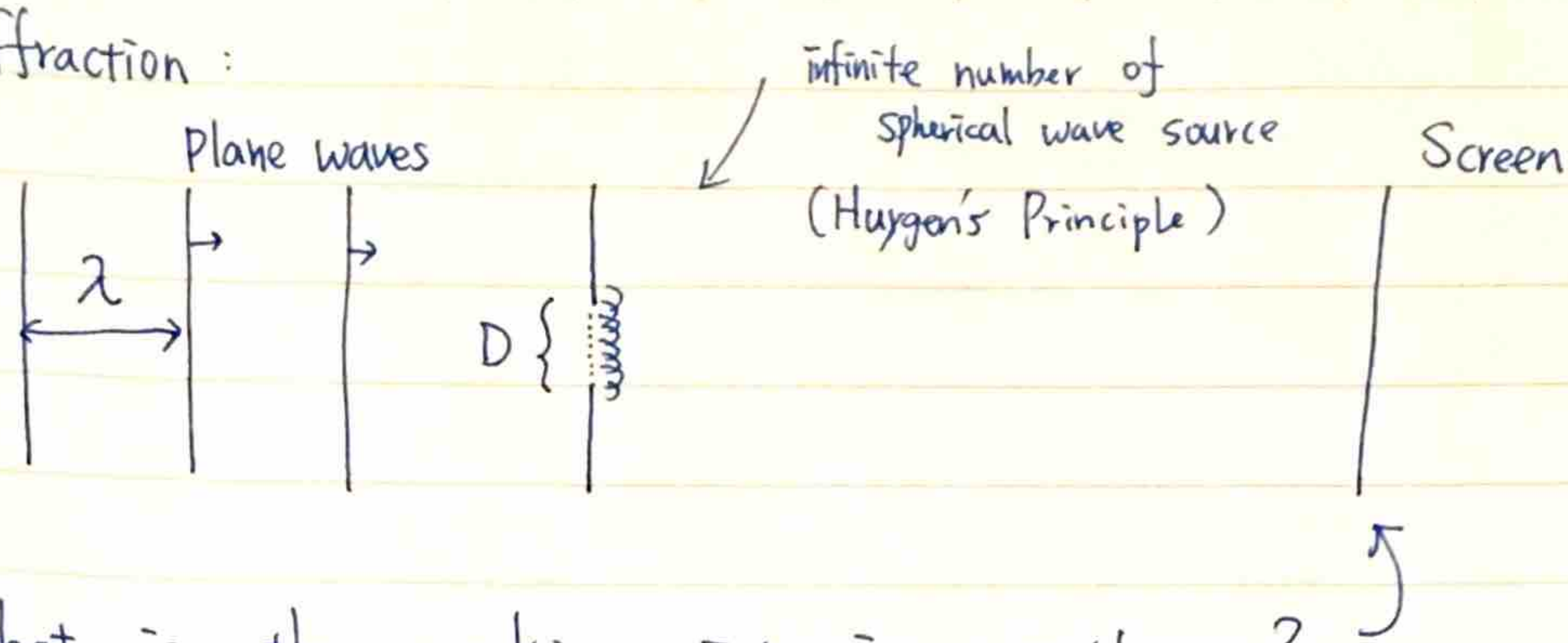
This situation: we will see the "interference" between all the spherical wave sources.

We call it "diffraction"

Feynman: No one has ever been able to define the difference between interference and diffraction satisfactorily

It is just a question of usage.

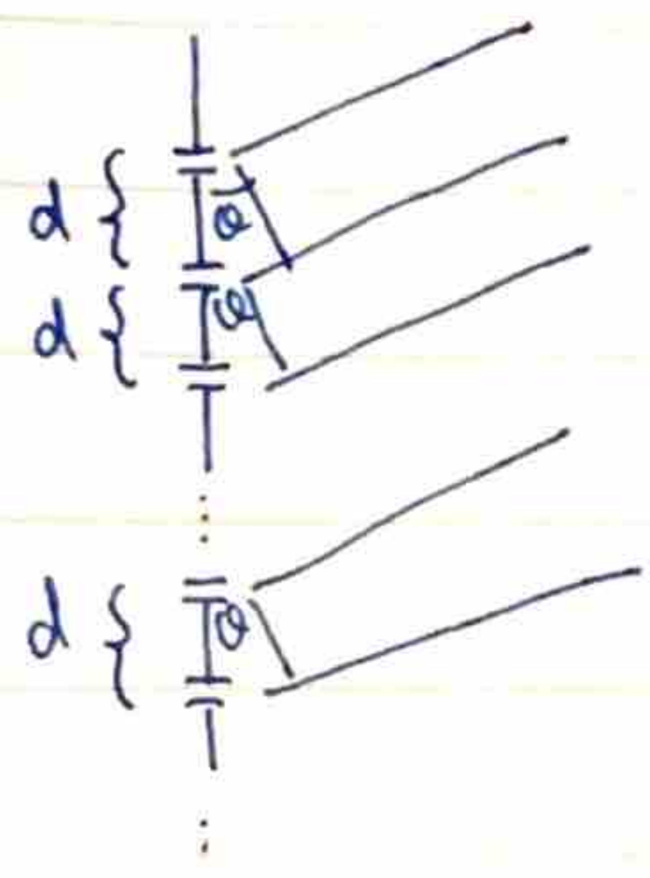
Diffraction :



What is the resulting intensity pattern ?

< Method I >

Reminder : N - slit interference :



$$\langle I \rangle \propto \left[\frac{\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \right]^2$$

Where δ is the phase difference between near-by slits

$$\delta = \frac{d \sin\theta}{\lambda} \cdot 2\pi$$

Consider the limit:

$$\begin{matrix} d \to 0 & & \delta \to 0 & & \\ N \to \infty & \Rightarrow & N\delta & = & \frac{D \sin\theta}{\lambda} \cdot 2\pi \\ Nd = D & & & & \end{matrix}$$

$$\langle I \rangle \propto \left[\frac{\sin\left(\frac{N\delta}{2}\right)}{\frac{\delta}{2}} \right]^2$$

We can define

$$\beta = \frac{N\delta}{2} = \frac{\pi D \sin\theta}{\lambda}$$

$$\Rightarrow \langle I \rangle \propto \left[\frac{\sin\beta}{\beta} \right]^2$$

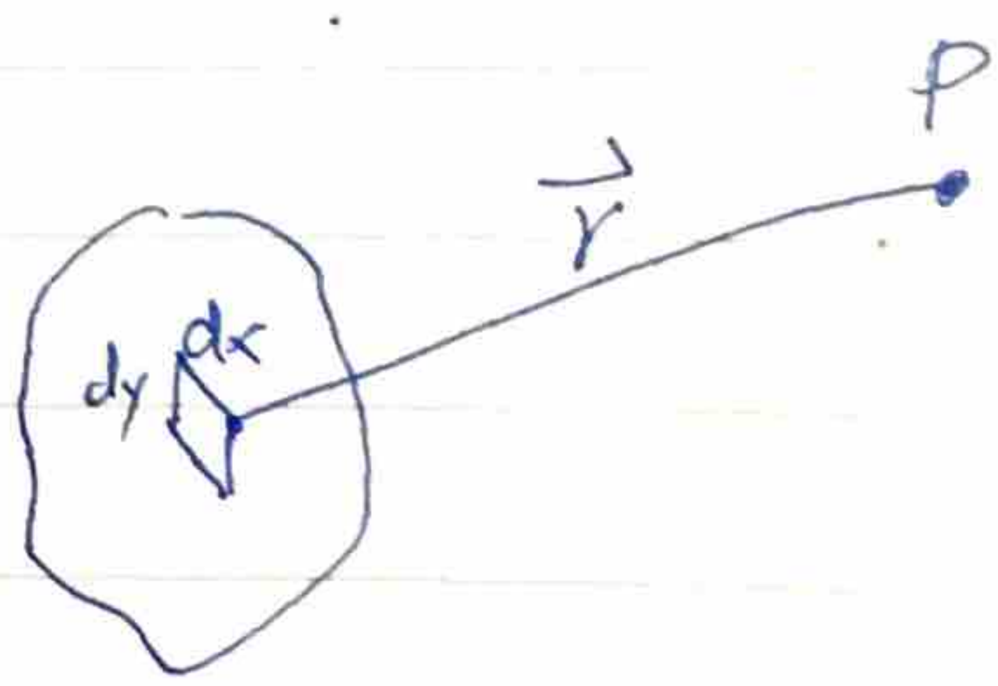
(Here we also assume that the intensity of individual point source $\propto \frac{1}{N^2}$)

⟨Method II⟩

Another method described in Georgi's book:

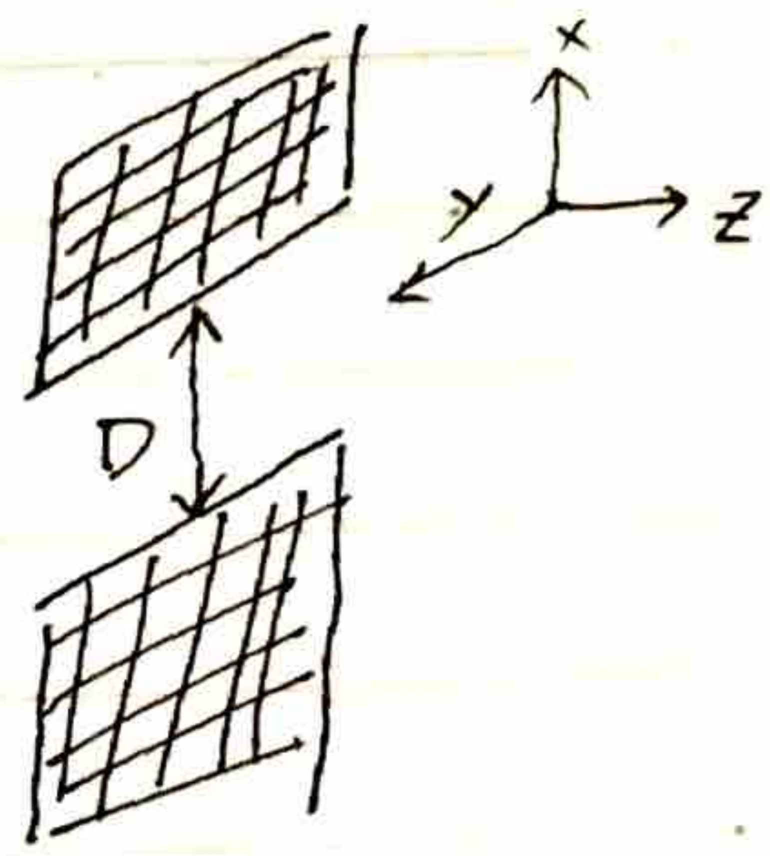
Do an integration over all point like source to calulate the total electric field.

$$\underbrace{C(k_x,k_y)}_{\propto \text{ Total Electric Field}} = \frac{1}{4\pi^2} \underbrace{\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy}_{\text{Unit area of the point-like source}} \underbrace{f(x,y)}_{\text{Shape of the Source}} e^{-i\vec{k}\cdot\vec{r}(x,y)}.$$



"Fourier transform of $f(x,y)$"

Let's consider a single slit experiment



$$f(x,y)=\begin{cases}1 & -\frac{D}{2}\le x\le\frac{D}{2}\\ 0 & |x|>\frac{D}{2}\end{cases}$$

$$\Rightarrow C(k_x,k_y)=\frac{1}{4\pi^2}\int_{-\frac{D}{2}}^{\frac{D}{2}}dx\, e^{-ik_x x}\int_{-\infty}^{\infty}dy\, e^{-ik_y y}$$

$$\boxed{\delta(x-a)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{ip(x-a)}dp}$$

$$=\delta(k_y)\frac{1}{2\pi}\underbrace{\frac{1}{-ik_x}e^{-ik_x x}\Big|_{-D/2}^{D/2}}$$

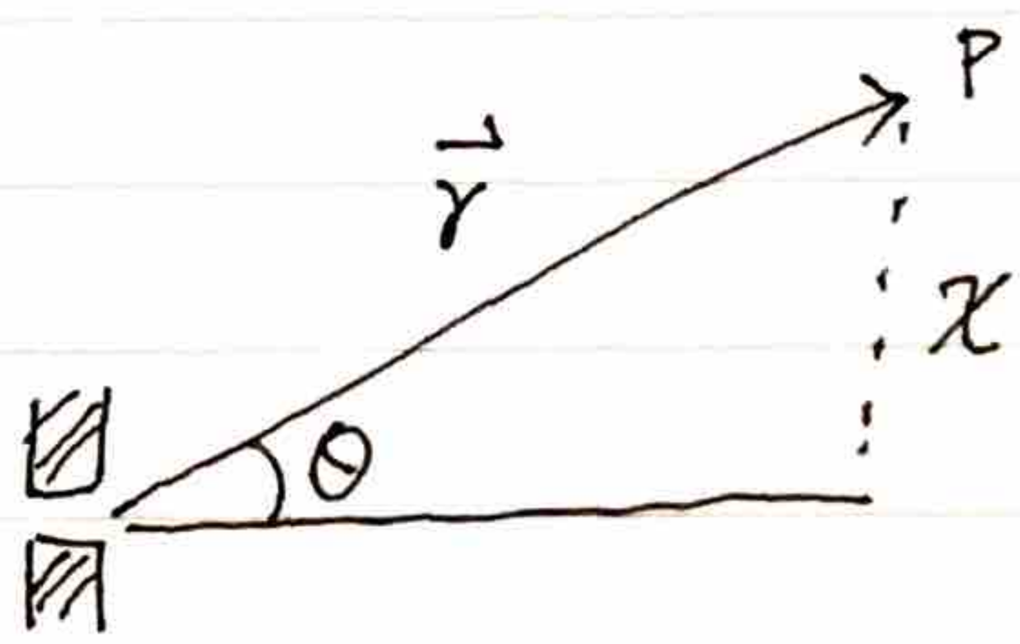
$$\frac{1}{-ik_x}\left[e^{-i\frac{kD}{2}}-e^{i\frac{kD}{2}}\right]=\frac{-2i\sin\frac{kD}{2}}{-ik_x}$$

$$=\frac{2\sin\frac{k_xD}{2}}{k_x}$$

Therefore

$$|\vec{E}|\propto C\propto\frac{\sin k_xD/2}{k_x}$$

$$I\propto|C|^2\propto\frac{\sin^2 k_xD/2}{k_x}$$



Since $\frac{x}{r}=\frac{k_x}{k}=\frac{k_x\lambda}{2\pi}=\sin\theta$

$$\Rightarrow k_x=\frac{2\pi\sin\theta}{\lambda}$$

$$\Rightarrow I\propto\frac{\sin^2\left(\frac{\pi D}{\lambda}\sin\theta\right)}{\left(\frac{2\pi\sin\theta}{\lambda}\right)^2}$$

Define $\beta=\frac{\pi D\sin\theta}{\lambda}$

$$\propto\frac{\sin^2\beta}{\beta^2}$$

(Same result as method I)

Red light 700 nm

$\lambda = D \sin\theta = \frac{D \cdot d}{r}$

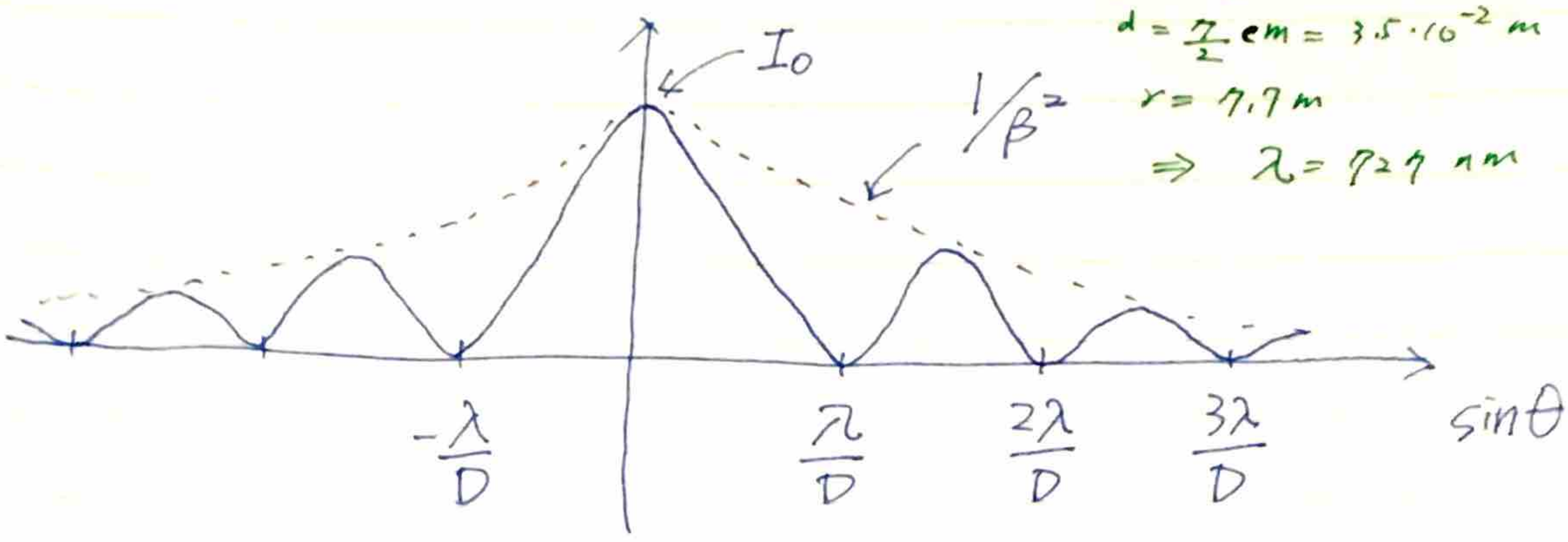
$D = 0.16\,mm = 0.16 \times 10^{-3}\,m$

$d = \frac{7}{2}\,cm = 3.5 \cdot 10^{-2}\,m$

$r = 7.7\,m$

$\Rightarrow \lambda = 727\,nm$

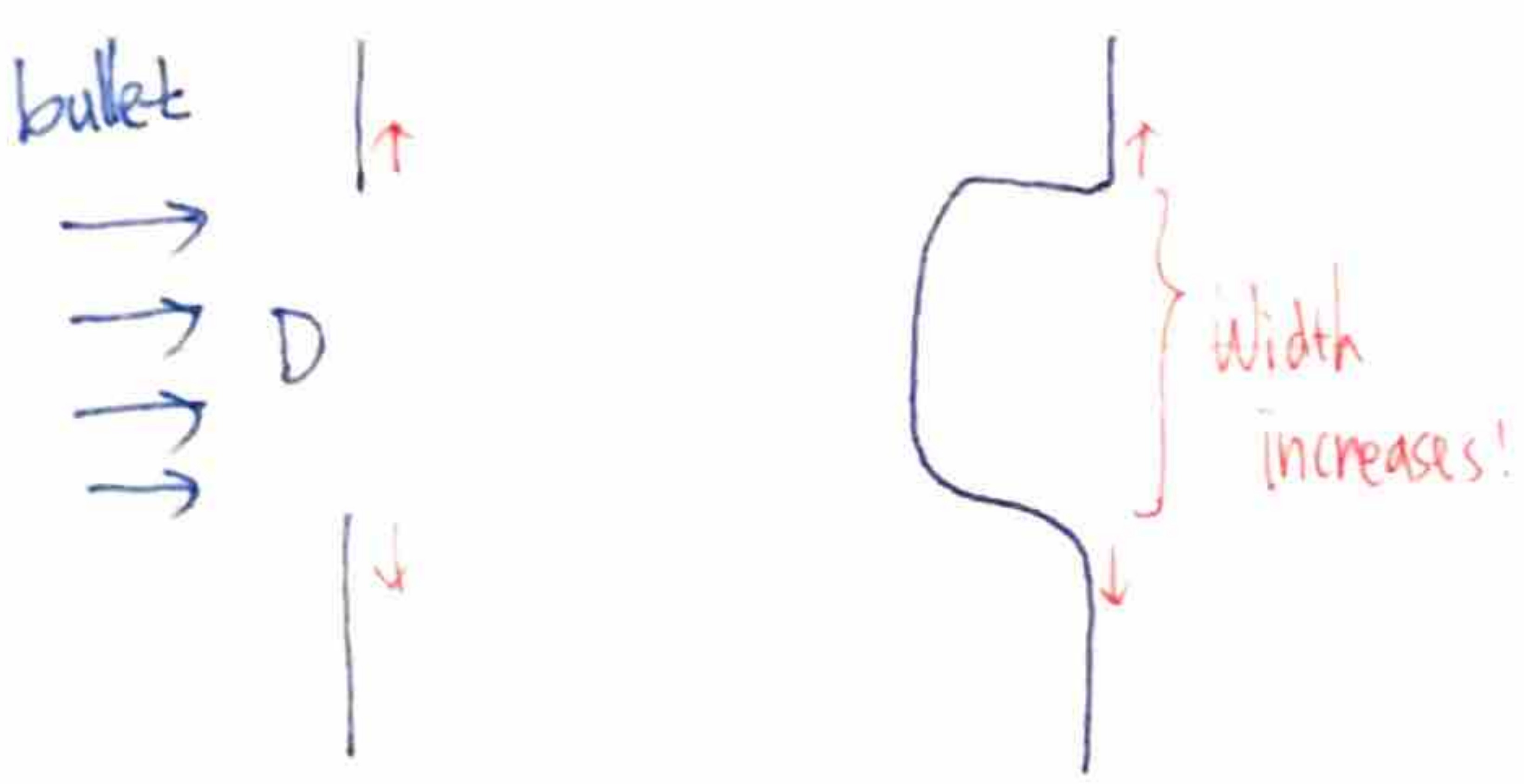
If we plot the result:



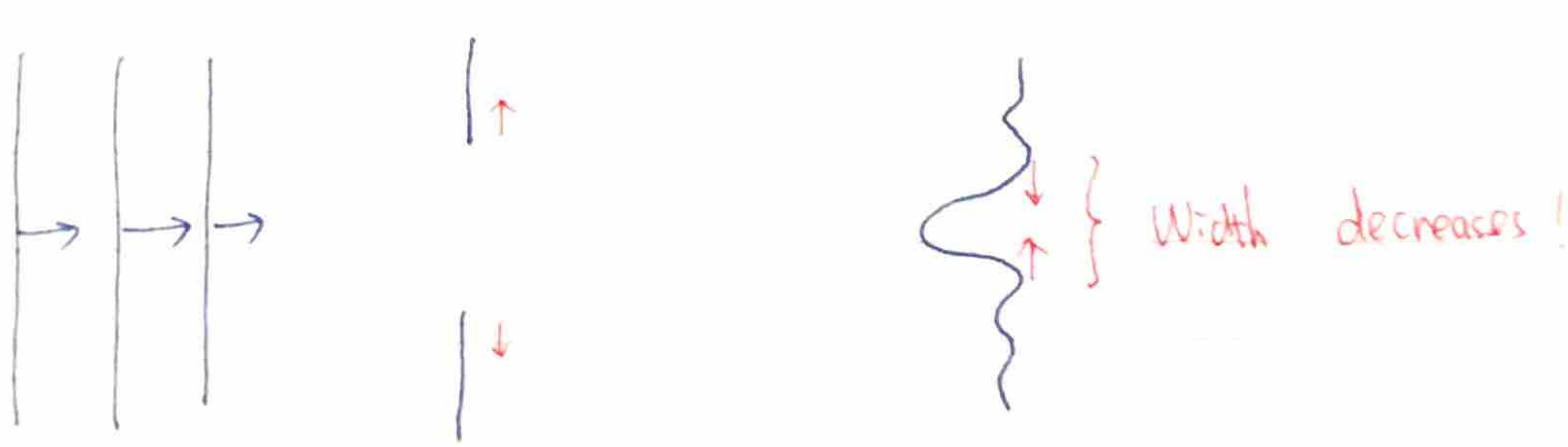
Observation:

(1) If we increase the size of the slit D:

$\Rightarrow$ the width decreases !! DEMO



EM waves:



② Distance between peaks $\propto \lambda$

Principle Maximum $\Rightarrow$ The width is larger for red light (longer wave length) than blue light (shorter wave length)

Slide

③ Intensity decreases quickly $\propto \frac{1}{\beta^2}$ as a function of β (or $\sin\theta$).

if D is large.

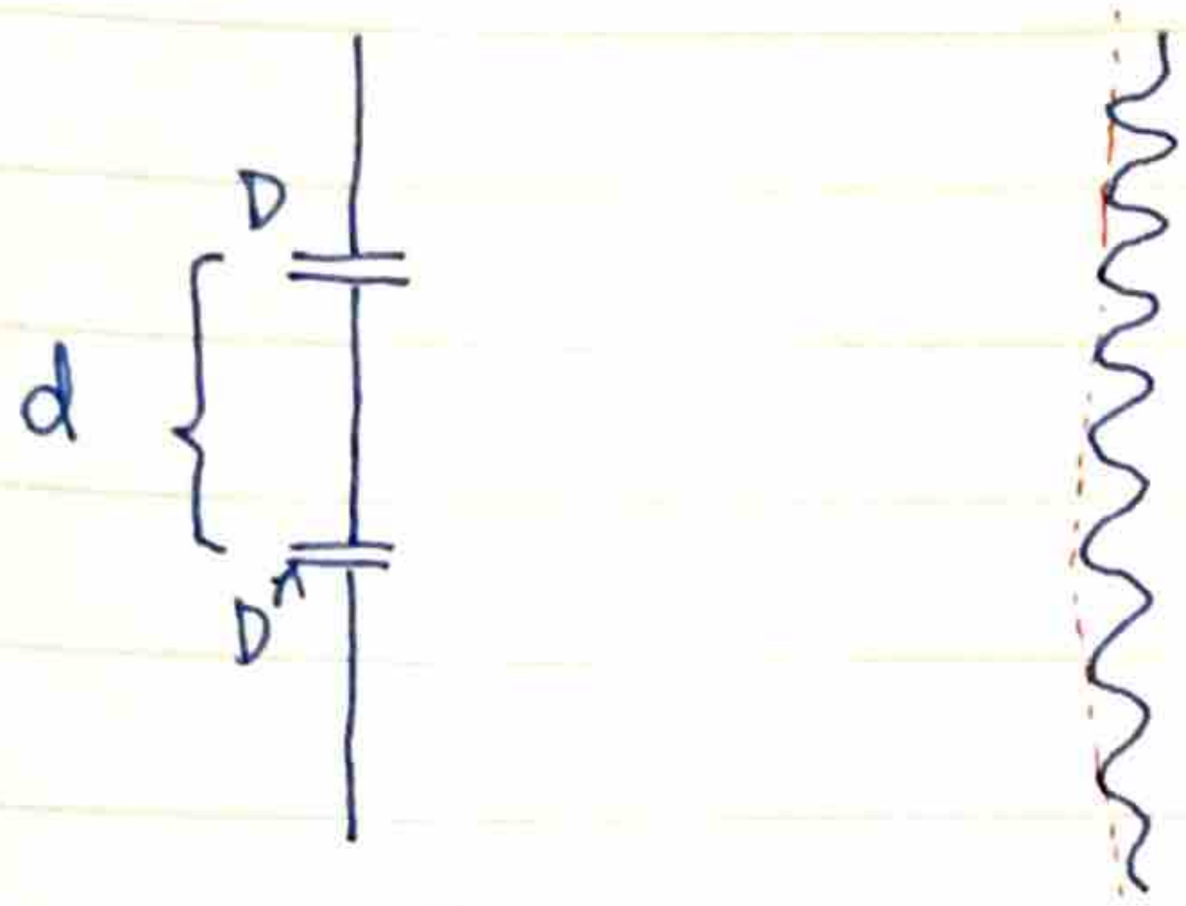
On the other hand: if D is smaller.

$\Rightarrow$ Intensity decreases slower.

Break ?

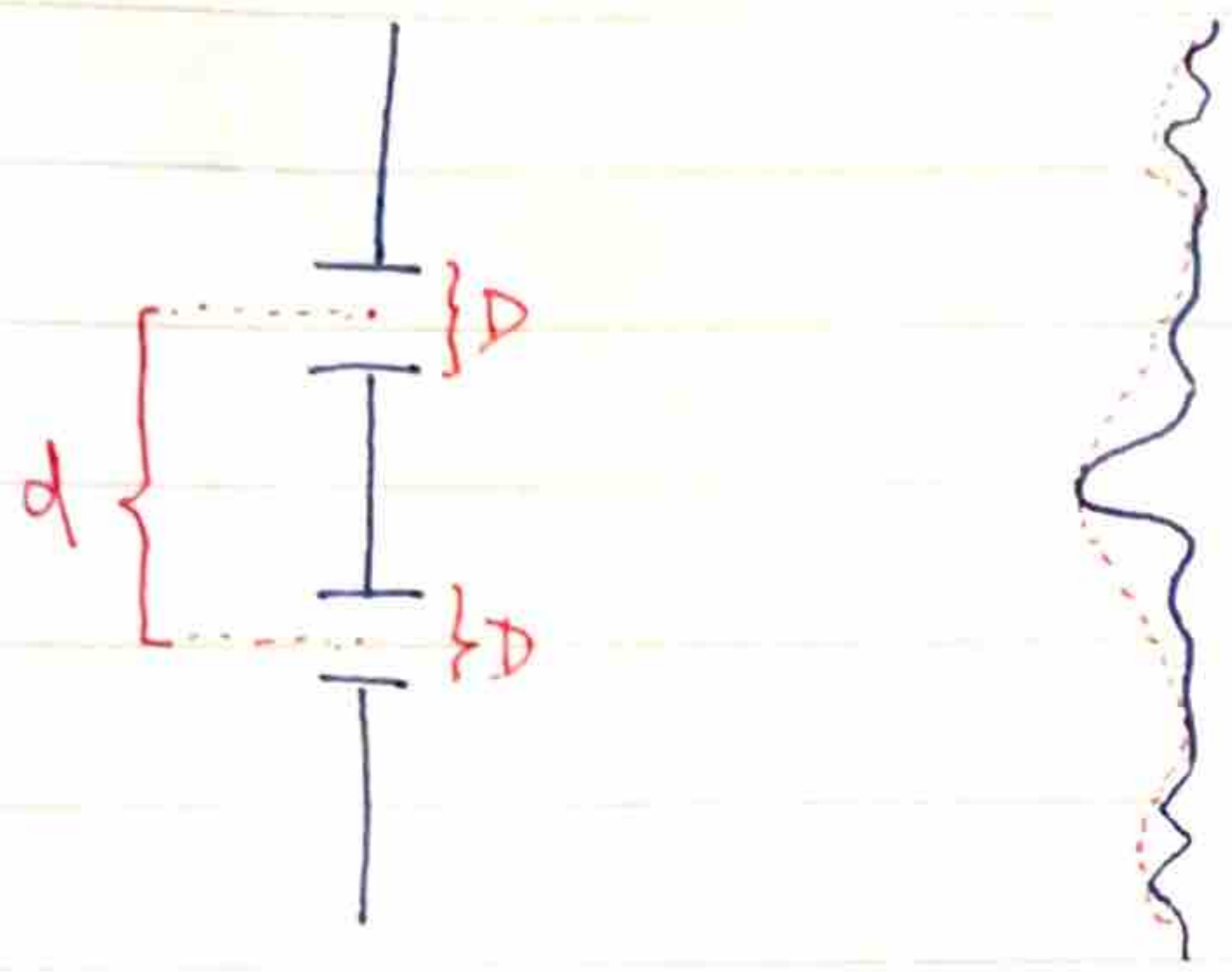
Coming back to the double-slit experiment:

Make it even more realistic: include the effect from finite slit width:



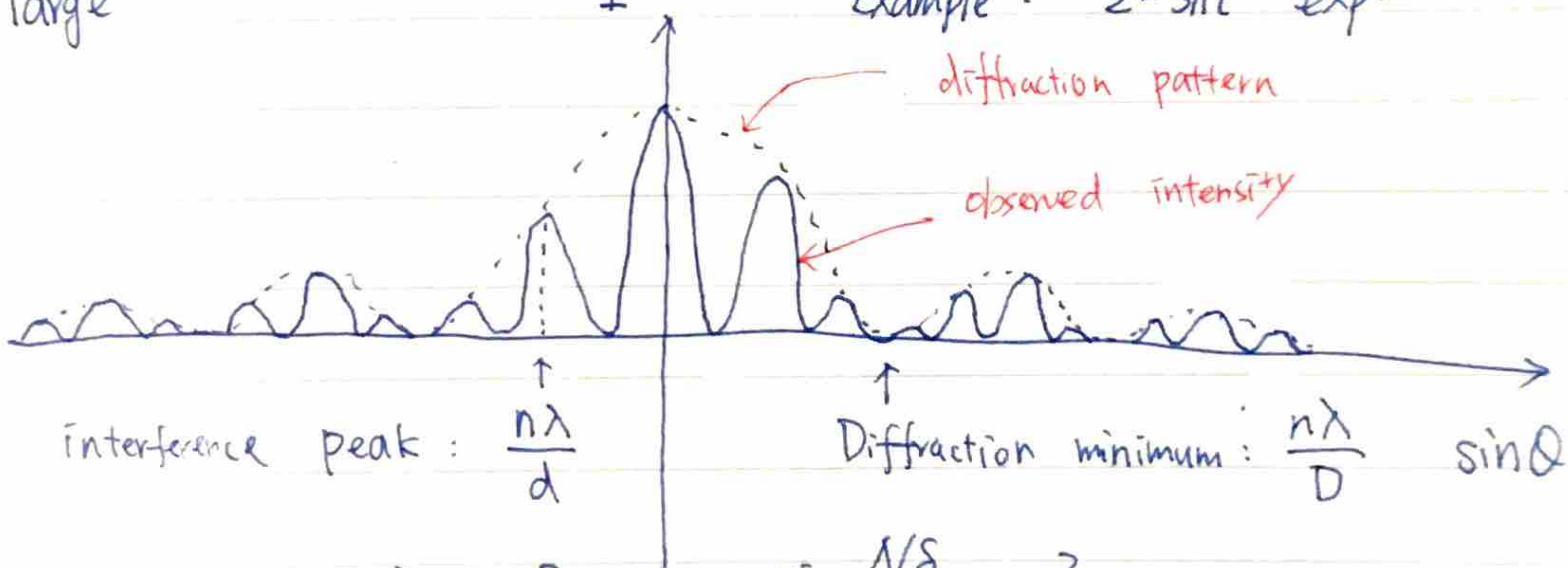
Multi-slit interference pattern modulated by the diffraction pattern.

D very small



D large

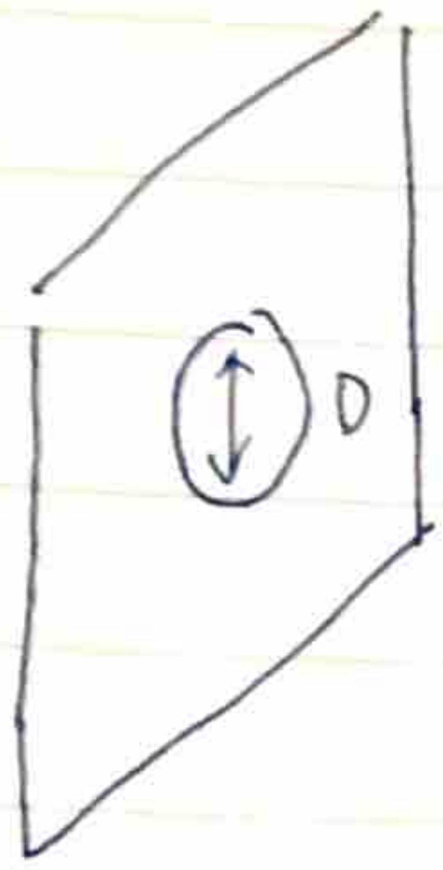
Example: 2-slit exp.



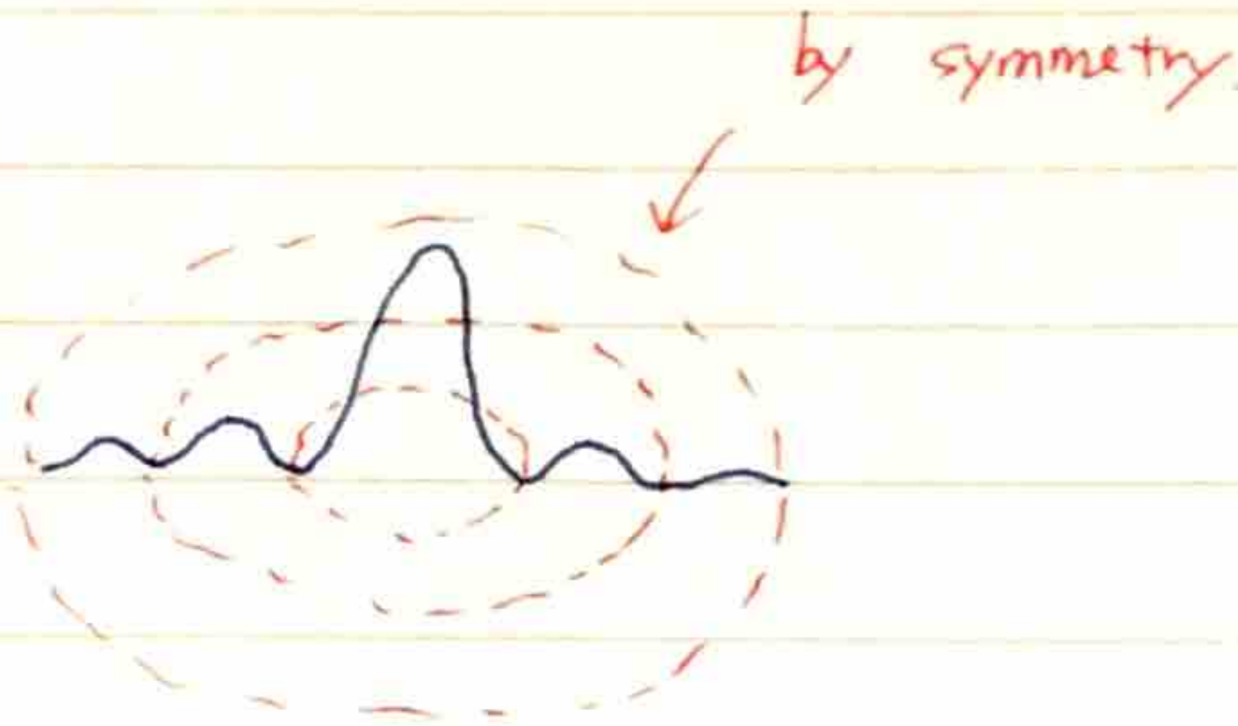
$$I = I_0 \underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{Diffraction}} \underbrace{\left(\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right)^2}_{\text{Multi-Slit Interference}}$$

$\beta = \frac{\pi D}{\lambda}\sin\theta$ $\quad \delta = kd\sin\theta = \frac{2\pi d\sin\theta}{\lambda}$

Let's consider a pin hole or aperture.



$\Rightarrow$



One can do the integration and we found that the intensity is :

$$I(\theta) = I_0 \left(\frac{J_1(\beta)}{\beta} \right)^2$$

$\beta = \frac{\pi D}{\lambda} \sin\theta$

J_1 : Bessel function of the first kind.

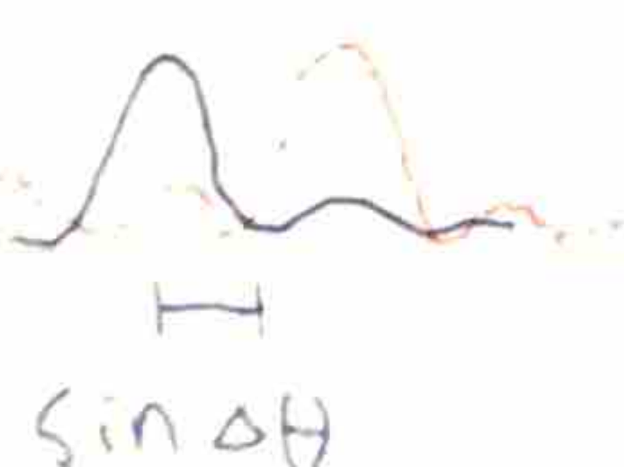
Solve : $J_1(x) = 0 \Rightarrow x \approx 3.83$

$$\Rightarrow \beta = 3.83 = \frac{\pi D}{\lambda} \sin\theta$$

$\boxed{\frac{3.83}{\pi} = 1.22}$

$$\Rightarrow \sin\theta \approx 1.22 \frac{\lambda}{D}$$

Resolution of a pin hole :



$$\sin\Delta\theta \approx \Delta\theta = 1.22 \frac{\lambda}{D}$$

such that we can separate the two peaks!

slide

Human pupil: 2 − 4 mm when narrow
3 − 8 mm when wide

Take visible light ~ 500 nm

$D \sim 5\ mm$

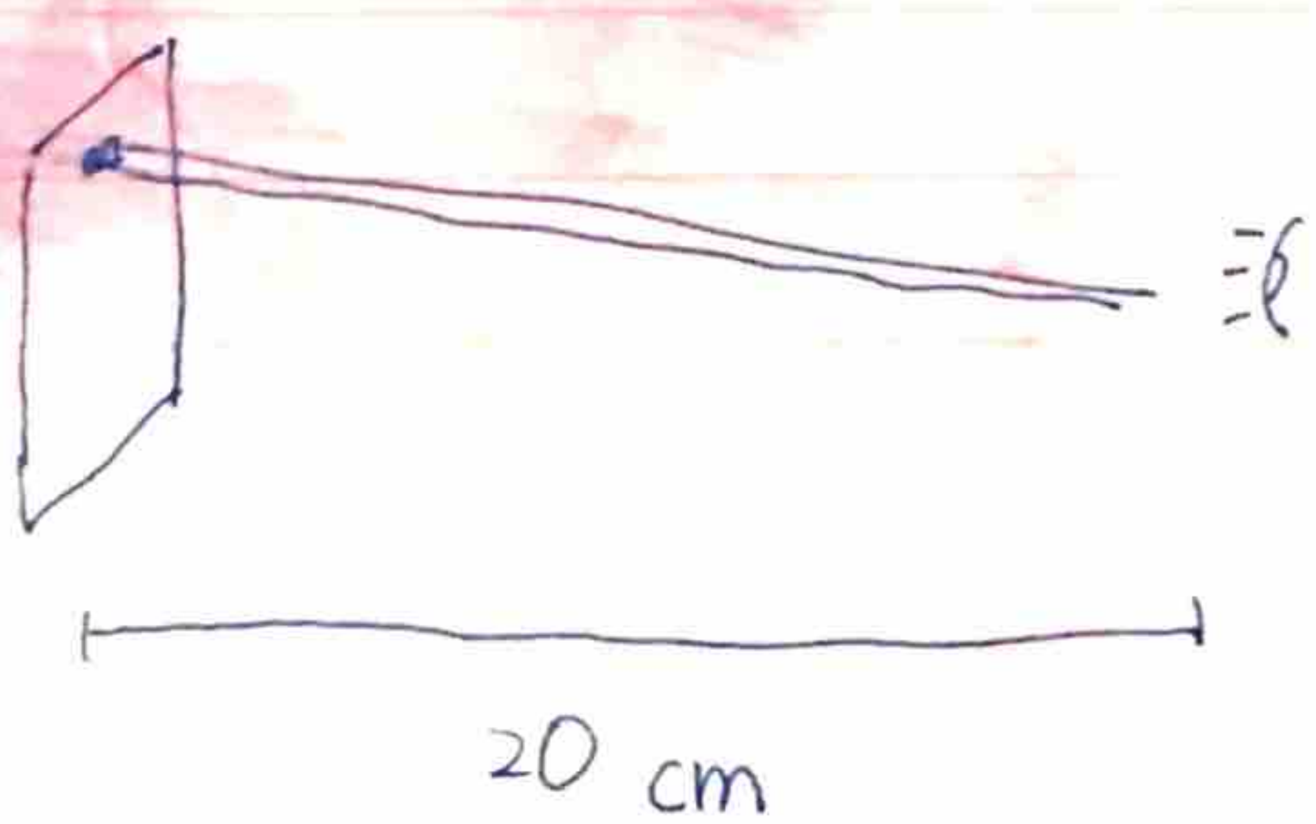
$$\text{Resolution} \sim 1.22 \frac{\lambda}{D} \sim 1.22 \frac{5 \cdot 10^{-7}}{5 \cdot 10^{-3}}$$

$$\sim 1.22 \cdot 10^{-4}$$

iPhone 7 : 401 ppi

$$\Delta x \sim \frac{2.54\ cm}{401} \sim 6.3 \times 10^{-3}\ cm$$

$$\Delta \theta \sim \frac{\Delta x}{20\ cm} \sim 3 \cdot 10^{-4}$$



Human Eye can resolve it!

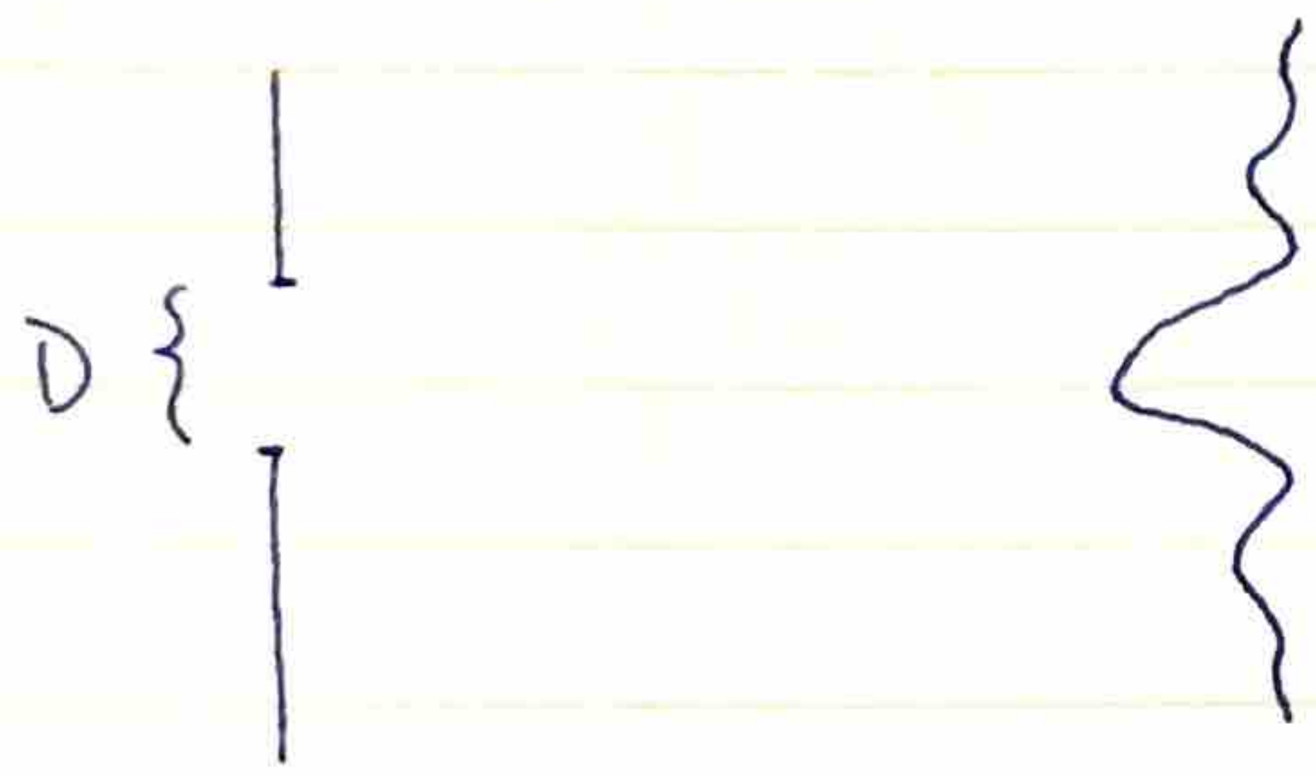
Will you buy iPhone X with 40000 ppi?

→ If Apple put 2000 pixels in 6 cm

~ the limit.

We have learned single slit diffraction

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \qquad \beta = \frac{\pi D \sin \theta}{\lambda}$$



This means that a laser pointer is not merely producing a pencil beam.

Suppose $\lambda = 500\,nm = 5 \times 10^{-7}$

$D = 1\,mm = 1 \times 10^{-3}$

Opening angle : circular source

$$\theta \approx 1.22 \frac{\lambda}{D} = 6 \times 10^{-4}$$

If we shoot a laser to moon : $L = 4 \times 10^{8}\,m$

Radius of the principle maxima : 240 km !!

$L \cdot \theta$