YEN-JIE LEE: OK. So welcome back, everybody, to 8.03. So before we start the lecture today, we will give you, as usual, a short review on what we have learned, and also, an introduction about what we are going to learn today. So last lecture, we were discussing an interesting phenomenon, which is seeing film interference pattern. As you can see from this slide, we were wondering why the soap bubbles are colorful.

And in the end of the class, we actually recognized that the reason why the soap bubbles are colorful is because of the interference phenomenon between the refracted light on the bubble. One puzzle path is that the light goes into the-- goes refracted directly from the surface of the soap film. The other possible optical path is to get refracted by the inner surface of the film. Therefore, the interference between these two paths actually created a colorful pattern on the bubble.

So we also learned about how thick is the soap film. And I think just a quick reminder, actually, we concluded that in order to see a colorful pattern, the thickness of the wall or, say, the film, should be something like in the order of 100 nanometer. So that's actually pretty remarkable, because that's already in the order of the size of a virus. OK. OK that's actually pretty cool.

So what are we going to do today? What we are going to do today is to continue the discussion that all kinds of different phenomenons, which can be explained by interference. We will learn interference phenomenon with a double slit experiment and using, for example, laser or water, and which I have a water tank here, which I will show you the interference pattern.

And also, the second thing we are going to learn today is how does a phased radar actually works. OK? So by the end of the lecture today, you should be able to learn why we should construct the radar in the way and how to actually focus on the electromagnetic wave to work one specific direction. So that's essentially what we are going to learn today.

The third goal is that we are going to make a connection to quantum mechanics from the lecture today. All right. So let's immediately get started. So before we start the discussion of a double slit experiment, I would like to remind everybody about Huygens' Principle, which you may already learned it from 8.02 or in the high school days. So what essentially is this principle?

So this principle is saying that if I take a look at all the points in the wavefront, basically, you can treat all those points on the wavefront a point source. And this point source, essentially, a point source of a spherical wave. And it's immediate from all the points on the wavefront. So you can see from this slide, basically, if we choose to focus on the yellow point on the wavefront, you can see that from each yellow point, you can actually treat that as a spherical wave point source.

And then what you actually need to do in order to calculate what would be the total electric field, for example, is to add up all those contribution from each point. And then you will be able to actually explain the interference pattern, which we see in the experiment. You may wonder where is this Huygens' Principle coming from? And although we are not going to derive that directly in the lecture today, but I can actually safely tell you that essentially, it can be derived from Maxwell's equation. OK? I will link some document, which actually shows the proof of the principle on the website and for your reference.

The other thing which you may or you may not know is that we are really lucky so that we can use this Huygens' Principle in our universe. Why is that? Because if you look at the mathematical proof of this principle, it is because the number of dimension, number of spatial dimension is odd, which is three in our universe-- Or in my universe also is yours, OK, [LAUGHS] happened to be yours, as well-- such that the Huygens' principle actually works.

On the other hand, if the number of dimension is even, there's no Huygens' principle, actually. So that's pretty interesting in that we are really lucky that it actually works in our universe. But I will not go into detail in 8.03. So let's get started with a concrete example, which we would like to further investigate to understand the interference phenomena. And those will prepare ourselves to the understanding of the design of the radar, for example.

All right. So suppose I have experimental set up here which contain a wall where on the wall, there are two slit, A and a B. The upper one is A. The lower one is B, as designed here. And from the left inside there's an insert plane wave with a wavelength lambda, which is showing here. And this plane wave, plane electromagnetic wave, or can be water wave, et cetera, essentially approaching the wall with these two slits there. And we were wondering what would be the resulting pattern on the screen.

This screen is actually pretty far away from the experimental setup, the wall on the left-hand side. How far is that? The distance between the screen, which shows the resulting interference pattern, and that the wall is actually defined. It's actually given here. It's actually called L, capital L. And in this experimental setup L essentially pretty, pretty large and is much, much larger than the d, where d, small d, is the distance between the two slits. OK?

So our job now is to understand what will be-- and to predict what is going to be the interference pattern coming from the electromagnetic wave which pass through point A and point B, and what is going to happen over, say, what would be the result which we will observe on the screen. OK?

So the first thing which we can do is that we can now assign observer, which is called P, one of the point of interest on the screen, which is located here. And then we can link or, say, the connect the point A, which is the location of the first slit and the location of the second slit, which is called B. We can link those points together by a line. And that is actually denoted by AP and the BP, these two lines.

Since we are talking about L, which is essentially very, very large, assuming that the distance, the length scale of the distance between the wall and the screen is much, much larger than the length scale of the distance between the two slit, which is d. Therefore, I can safely assume that AP and the BP are almost parallel to each other. Right? And I can also try to express the location of the P point by using the angle between BP and the horizontal direction. OK? And the horizontal direction is actually showing there's a dash line here. And the angle between BP and the horizontal direction it's called theta here.

OK. So since AP and the BP are almost parallel to each other, I can now calculate what would be the optical path length difference between AP and the BP. Right? So in order to actually calculate the phase difference between the electromagnetic wave coming from slit A compared to slit B, I need to calculate-- again, like what we did last time-- optical path length difference. OK?

In this case, I can call the distance between A and P, rA. And then I can also call the distance between B and the P, rB. Then the optical path length difference is called rB minus rA. And then we can actually calculate that because we have already given you the angle between BP and the horizontal direction.

And basically, we can safely conclude that the path length difference is actually this line here. Therefore, I can actually calculate and get the optical path length difference, the difference between rB and the rA to be d sine theta. OK? Once we have that, it's actually pretty straightforward to calculate what would be the phase difference.

The phase difference between the field coming from slit A, which I will call it EA here, and the field coming from the slit B, which I will call it EB here. The phase difference, as you define a lot of time to be delta, delta can be calculated by the optical path length difference, d sin theta, divided by lambda, which essentially telling you how many period have passed when the light have to actually overcome this-- or say have to pass through this optical path length difference.

And, of course, these things need to be modified by 2 pi in order to translate from a number of period to a phase difference. Therefore, you get the phase difference between AP and BP to be delta equal to d sine theta divided by lambda times 2 pi. OK. So you can see that all those calculations are pretty straightforward. Maybe you have already seen that before in an earlier class.

But what I want to say is that it is actually because of Huygens' Principle, such that you can't expect something which will show up at point P, right? If you don't have Huygens' Principle what is going to happen? What is going to happen is that the light passing through this slit will just go straight. And they will never overlap each other. OK?

So that's actually why, because of the Huygens' principle, all the points on the wavefront are treated as a point source of a spherical wave. OK? So that is essentially why you can expect that something will hit the P point, which is because, in this case, we have two points, two point source. And they are emitting spherical waves coming from these two points. OK? So it is really because of Huygens' Principle, which applies here, such that we can actually observe the phenomenon at the P. And now, we have managed to calculate the phase difference, which is delta, presented here.

So what the next question is, what would be the intensity? Since we have already calculated the phase difference delta, what would be the intensity observed at P? So for that, we have already prepared ourselves from the last few lectures. So now, we can actually calculate what would be the total E. The total E will be equal to EA plus EB. And here, I'm going to use complex notation just for simplicity.

And basically, you can rewrite EA and the EB as E0 exponential i omega t minus k times rA plus E0 exponential i omega t minus k rB. The first term is actually telling you the contribution from the first slit, slit A. And the second term is actually telling you the contribution coming from slit B.

In this set up, I'm telling you that I have the plane wave coming from the left hand side of the experiment and actually hitting the wall. And you can see that from the drawing. Actually, the wavefront, essentially, actually telling you that the direction of the electric field is actually in the Z direction in my coordinate system shown on the board. So basically, the Z direction is actually pointing to you guys. And that means the electric field is actually oscillating in this direction. OK? So therefore, I have to be careful of those vectors. So therefore, I need to give it other direction. And in this case, it's actually the Z direction.

And also, you can see that the amplitude is actually denoted by E0 because I always assuming that both slit have the same finite width. For the moment, ignore the width of the slit. And also, they are coming from the same plane wave. Therefore, the amplitude is all denoted by E0. OK?

So now, I have the expression here. And I can now go ahead and simplify this expression and rewrite that in this form. So I can now extract the E0. And also, I extract the common factors here, which essentially the exponential i omega t and also, minus k rA. I can actually factorize some part of the exponential function out.

So the choice I made is that I actually could factorize out exponential i omega t minus k times r. Basically, I take these out. And I get this term showing here, omega t minus k rA. I take this out. Then basically, what you are doing to get inside will be 1 plus exponential minus i delta, actually. times z. OK?

Why is that delta? Because once you factorize out or take out exponential i omega t minus k rA, basically, you are left with something proportional to exponential i minus k rB minus rA, right? And that is actually the optical path length difference here. And also, of course, you can always rewrite lambda over 2 pi, right? Basically, you write this to be k times d times theta. Right? So therefore, you can actually immediately identify the second term is to essentially exponential minus i delta. OK?

Any questions here? OK. Because d sine theta essentially is just rB minus rA, therefore, I safely replace that by delta here. OK? All right. So since everybody's on the same page, I can now, again, factorize out not only the omega t minus kA term, but I can actually do a trick to factorize out, also, exponential minus i delta divided by 2 out. And basically, what I'm going to get is exponential i delta over 2 plus exponential minus i delta over 2.

This reason why I'm doing this is because, huh, now, I have this term identified. And this is actually just 2 times cosine delta divided by 2. All right? OK? So now, I'm really pretty close to the intensity.

So what would be the intensity coming out of this electric field? That is actually going to be average intensity, as we discussed last time in the lecture. The average intensity is proportional to square of E vector. Right? In the complex notation, how do we evaluate the absolute value of E vector square? In the complex notation, basically, you get basically, E times E star, where E is actually the amplitude, which is the size of the E vector, the magnitude of the E vector.

Then, basically, you will see that this will be proportional to cosine square delta divided by 2. Right? Because you can see that if I calculate EE star, then all the terms with related to exponential i something actually got cancelled. Right? So therefore, you can see the "aha" very, very quickly. We can show that the intensity will be proportional to cosine square delta divided by 2, where delta is the phase difference between the first path and the second path. OK? Any questions so far? OK.

So we can see that the intensity essentially changing really rapidly as a function of delta. Right? So when I have a situation where delta is equal to 0-- let's actually stop here a bit and enjoy what we have as you learn from here. All right? So if you have delta equal to 0, what does that mean? That means there's no phase difference between the first and second electric field.

Therefore, when you add them together-- just a reminder about the notation we were using before. So if you draw the vector in a complex frame, what you are doing is that you are actually adding EA and the EB together in the most efficient way, right? Because the delta is equal to 0, the phase differences is equal to 0. Therefore, you are actually adding them in a straight line. OK? So that actually will give you the maxima intensity. Because when delta is equal to 0, cosine 0 is 1. Right? Therefore, you are reaching the maxima in the intensity.

So now, I can always increase my delta until a number which is actually pi. What is going to happen is that if I still use the notation which I was using for the complex frame, what it does this is that, huh. Now, I am actually completely cancel the electric field, because the phase difference now is pi, right?

So therefore, in the complex frame, you are adding the two vectors in way such that they completely cancel each other. The magnitude of the two vectors are the same, as shown here, which is actually E0, right? Therefore, what you are going to get, as you expect, is going to be 0, because they completely cancel. OK? You can also see that from this formula we did right here. When delta is equal to pi, then essentially, cosine pi over 2. Then you get intensity equal to 0. OK? Everybody accept this? All right.

Now, I can still continue and increase the delta, for example, until delta is equal to 2 pi. Then you are getting this again. Basically, you have EA and the EB, again, line up each other. And the difference is that this EB actually rotated maybe 360 degree. And basically, you will see that, again, the intensity become the maxima again. OK?

So that is actually how we can actually understand this result. And, of course, you can also go ahead and plot or simulate this result in the computer and really draw the amplitude, really draw the intensity as a function of angle here, or, say, the delta here. As you can see from here, that the intensity is actually reaching the maximum in the center.

Why is that? In the center, if I have observer here in the center, what is going to happen is that the path length, optical path length between AP prong and the BP prong is going to be the same by symmetry, because it's actually in the optical center. Therefore, you will expect that delta is actually equal to 0. OK? So that's essentially why you see the maxima there.

And if you start to move away from there, you will see that the delta start to increase. And at some point, you'll reach a minima, which you can see that on the plot. And that is actually because now, due to the increasing optical path length difference and the phase difference, the two electric field is starting to cancel each other, which actually produce the black pattern there.

And finally, after it pass delta equal to pi, then these two electric fields start to work together again. All right? They're collaborating again. And you can see that again. You would get another maxima afterward. OK? And here, you can see that is actually my calculation. And, of course, I can do a demonstration to you to really show that this is actually what we are going to see based on the demonstration we are going to show here. So now, I am going to turn the light off. And here, I have a device which actually contain a water tank. And I need to actually turn this thing up.

On the water tank I have two vibrator, which is actually acting as a point source. So basically, those vibrator vibrating up and down to create waves in this tank. OK? So basically, you can see that, huh, really, you have two point-like source. And you can see spherical waves is actually really generated and is really propagating away from the point source. OK?

And what I can do now, you can see that this picture is really dynamic, because we can see that wavefront essentially moving as a function of time. So what I'm going to do is to really change the frequency of the light, which is actually shining on this water, so that you can actually see the fixed pattern here. And now, I am going to change the light frequency. You can see now I only shine the water tank at the specific time which match the speed of the propagation of the water wave. And you can see, aha, I've actually managed to freeze the wavefront. We see? OK.

So you can see, now, really, you can see coming from the source, they are circular wavefront, which actually mimicking the result from Huygens' Principle. And you can see that they are complicated interference pattern forming. You can see that at some point they have constructive interference. If you focus on the central part, you can see that the maxima is actually reach there.

On the other hand, if you move away, a little bit away from the center, you can see that really, the intensity drop. And at some point, you will also see that, OK, again, I am changing the procedure in such that the phase difference between the contribution of our source A and the B essentially equal to 2 pi. In that case, you will be able to see that another maxima is actually created again.

So now, we can actually also show you that a lot, in fact, based on this glorious pattern, let's actually take a look at the projector here. So if I look at on the individual slide, which I have here, you can see that those are actually a point-light source and is creating a circular pattern. And now, I can actually overlap with two patterns together. And you can see that when I have the center of the two circles pretty close to each other, you can see that really, you have very small d.

In this case, you have very small distance between source number one and number two. Then basically, based on our expression, so you can see that delta is equal to d sine theta divided by lambda times two pi, right? And you can actually calculate sine theta will be equal to delta divided by k times t. OK? When delta is equal to pi, that is going to give you a minima where, essentially, also showing here, the minima is shown as the black pattern here. OK? You can see from on here.

So what this says, your formula is showing you that when I have d, which is very small, what is going to happen is that I'm going to get sine theta to be very large when d is actually very small. And that can be shown here. When I have d, which is the distance between the center of these two point source, very small, you can see that the place you get the minima is really far away from the center, which is actually here. OK?

Now, what I'm going to do is to increase the distance between these two source. According to our position, what is going to happen is that the central maxima will decrease. The position where you get a minima will be moving closer to the center, according to that formula, because it's proportional to 1 over d. And we can do this really carefully to see if I can succeed.

And you can see that really, when I am moving these two slides away from each other, you can see that the pattern is changing, right? And the center maxima, or, say, this Gaussian-like curve there becoming narrower and narrower. OK? So that essentially what we can actually observe form here. And our calculation really works very well here. Very good. So do we have any questions regarding the demonstration we have here?

OK. So all those things seems to be pretty straightforward to you. And what we are actually now is seeing a position where we can actually discuss how we actually can understand the radar, which is how actually radar works. So here is actually how radar works.

Suppose you have some unknown object, which is like an airplane, OK? And you would like to know where is this object. What you do, actually, is to shoot whatever radio waves toward some direction and see if there are something coming back. Right? Then you know there's something on the sky because you can detect the refracted wave. Right?

So we shoot this airplane. And then something is going to come back. And now, we can say OK. In that direction I have something coming back. That means there's something there. And I can also measure the time it takes for the wave to come back. Then I know where it's actually that object. Right? So that's actually a pretty straightforward thing to do.

However, there's one difficulty. So this is actually the radiation pattern of oscillating dipole which we actually learned before. So the problem is that, OK, what we really need is electromagnetic wave, which is actually very, very narrow in angle and pointing to some specific direction. And then I would like to see if I can get some refractive wave coming from that direction. OK?

The problem is that, look! if I oscillate some charge up and down, the radiation I'm getting is really, really broad. Right? So it's going toward all kinds of different direction. So if you use this to detect things, you are always going to get something coming back, because it's actually shooting the electromagnetic wave into random direction. And you are not sure any more where is actually this object you are trying to detect. OK?

So that's actually apparently a problem. And what we can actually do is to make use of the interference phenomenon, which we can actually learn from here to actually try to make sure that the electromagnetic wave is actually pointing to some specific direction we want. So let's actually go ahead consider a three slit experiment.

I have this setup changed. Originally, I have two slits. And now, I drew it in three holes on the wall. And, again, I have the distance between the slits to be d. And I call this slit number 1, 2, and 3. And we were wondering what would be the interference pattern on the screen, which is actually far away from the wall, as a distance of L. And I'm interested in their intensity at the point P on this screen. OK?

So what I am going to do is to basically repeat what we have done in the previous example. I'm trying to connect 1 to the P, 2 P, and the 3 P, basically, connect the slit to the point of interest on the screen. And I can actually also-- you know this angle, this 1 P to the horizontal direction, this angle is called theta in my notation. Then clearly, I can go ahead and calculate what will be the optical path length difference between of the light coming from slit number 1, slit number 2 and the slit number 3. OK?

And in this case, what I'm interested is delta 1, 2 and delta 1, 3. Right? Since the screen is really far away from the wall, therefore, I can actually savor the assurance that these two angle is actually theta because the three lines, due to the large distance, this L is actually really, really large. Therefore, they are actually almost parallel to each other. OK?

So what is going to happen is that delta 1, 2, which is the phase difference between light from the first slit and second slit, is actually going to be equal to delta 2, 3. It's going to be equal to the phase difference between the second slit, the light from second slit and third slit. And what is actually that number? This number is going to be equal to d sine theta divided by lambda times 2 pi. It's exactly the same as what we actually get from the first example. OK?

Therefore, what is going to happen is that no matter what theta I choose, the phase difference between nearby slit is actually a constant, which is actually this one. And I will call this phase difference to be delta. I would like to ask you a question now. The question is, how do we choose the delta here such that I have completely destructive interference?

Now, I have three vectors, vector E1, vector E2, and the vector E3. The phase difference between E1, E2, and E3, the nearby phase difference is actually delta. So the question is, how do I actually completely cancel the electric field so that I have completely destructive interference? Can somebody help me here? The hint is that you can actually use this vector sum idea in the complex frame.

STUDENT: [INAUDIBLE]

PROFESSOR: Yes, very good. To form a triangle in the complex frame, right? So what we can do is now choose the phase difference delta to be such that E1, E2, and E3 actually form a triangle. You see what I mean? Therefore, you can actually already get what would be the required delta value. The required delta value is going to be 2 pi divided by 3. Right? OK? So very good.

So now, we are not afraid anymore. So how about four slit experiment? I just add another slit, d essentially the distance between the fourth slit and the third slit. What will be the delta required to have destructive interference? Anybody can help me?

STUDENT: [INAUDIBLE]

YEN-JIE LEE: Very good. So if you have four slit, based on this intuition, which we developed from the complex notation vector sum, what is going to happen is that if you have four slit, the delta will be equal to 2 pi divided by 4. OK? So what does this tell us? So remember, the sine theta, sine theta is telling you the location where you get the minima. OK? So this is actually the power profile, or, say, the intensity profile. OK? And this is actually equal to 0. And this is actually delta. OK?

The place which you get zero intensity is actually becoming closer and closer to zero. Right? Because sine theta, which is the angle between horizontal direction and this observer P, is proportional to delta. When you have destructive interference at angle which is smaller, smaller, and smaller, that means what? That means the central Gaussian-like structure is going to be becoming narrower and narrower.

Does that make sense? Very good. So at least we found something interesting now. That means, ha, one idea to get very narrow electromagnetic wave pointing to some direction is to have a huge number of point light source and slit experiment such that I can actually construct something which is actually very narrow in angle. And I can use that to shoot the object which I would like to detect. You see what I mean? Does that make sense? OK?

All right. So that's very good. So now, let's actually consider an N slit interference pattern OK? So suppose, now, I have not only 1, 2, 3, and then many more until N slit. All right? I can now go ahead and calculate the E total, which is the total electric field coming from all of the slit we have. Basically, this will be equal to E0 exponential i omega t minus kR where I define r1 is roughly capital R. OK?

That's essentially the contribution from slit number 1 OK? And this contribution from slit number 1 is going to be looking like exponential i omega t minus kR minus delta, right, because there is a phase difference between the light coming from first slit and the second slit, which is actually delta. All right?

So what would be the third term? So these actually coming from slit number 2. What would be the third term? Exponential i omega t minus kR minus what?

STUDENT: 2 delta.

YEN-JIE LEE: 2 delta, yeah, because you can see that coming from here, seems the distance between theta as constant, which is d. Therefore, the phase difference between nearby slits is actually a constant. Therefore, I accumulating the phase difference now. I get 2 delta here.

And this is actually a contribution from the first slit. And the et cetera, et cetera, until the Nth slit, which is actually going to be exponential i omega t minus kR minus N minus 1 delta. And summing all those things together, and all of them are in the Z direction. OK?

So I'm now going to calculate this dimension. So basically, you are getting E0 exponential i omega t minus kR. I can actually factorize these factor out. And what am I going to get is 1 plus exponential minus i delta plus exponential minus exponential minus i 2 delta plus blah, blah, blah. And basically, you will get exponential minus i minus 1 delta in the first term. And all those things are pointing to the Z direction.

And this, I know how to actually calculate. Right? Just a reminder, basically, if you calculate summation N equal to 0 to N minus 1 r to the Nth. And these will give you 1 minus r to the N divided by phi minus r. OK? So basically, I can now go ahead and calculate this.

And this will basically give you 1 minus-- OK, so the small r here has been replaced by exponential minus i delta, right? So therefore, what I'm going to get is 1 minus exponential minus i delta N for the upper part. And then I have 1 minus exponential minus i delta in the lower part. OK? So that actually make use of this formula, which are here. And, again, it should be simplify these series. All right?

As usual, what I'm going to do is to use the trick similar to what I have done there to actually get cosine function out of the exponential functions. All right? So what I'm going to do is to factorize out exponential minus i delta N over 2 for the upper part. So basically, I get exponential minus i delta N divided by 2, exponential i delta N divided by 2 minus exponential minus i delta N divided by 2. OK?

This is actually divided by exponential minus i delta over 2 exponential i delta over 2 minus exponential minus i delta over 2. All right? The reason I'm doing this is because I would like to actually make this a cosine function. OK? Any questions so far? OK. So if no question, then basically, this expression can be, again, rewritten as exponential minus i delta N minus 1 divided by 2, because I have this denominator nominator exponential i delta N over 2 and that exponential minus i delta divided by 2. OK?

Therefore, I can combine them all together and then get this expression here. And this is actually exponential minus exponential. Therefore, I am going to get sine out of it. And basically, I get sine and delta divided by 2 divided by sine delta over 2.

OK. So now, I can actually go ahead and calculate what will be the resulting intensity. Right? The resulting intensity is going to be proportional to the square of the electric field. Right? So basically, the intensity will be proportional to E square. And that is actually equal to E times E star.

E and the E is a complex conjugate. And basically, you will see that this will be proportional to sine and delta divided by 2 divided by sine delta over 2 square. Therefore, the intensity will be equal to i 0 times sine and delta divided by 2 divided by sine delta over 2. And then square that. Any questions?

So after all this work, we have arrived at expression which is very hard to understand. Right? [LAUGHS] So what I'm going to do to help you is to really plot the result as a function of delta on the screen. You can see there are four plots here. The first one is N equal to 3. The upper left one is N equal to 3. So you can see that the pattern looks like this. So at delta equal to 0, surprise nobody, you are going to get maxima. Right? Because delta is equal to 0, you are adding N vectors the most efficient way. Therefore, you are going to get the maxima, which is i equal to i 0. OK?

And if you move away from the center, delta equal to 0, and you see that is a small bump in between. Then you can continue and continue. And you see that there's another big peak again. You see? So that's essentially the structure if you plot this result, i equal to something proportional to sine square this expression there. And that's essentially what you will get when N is equal to 3. OK? And this is essentially how I remember this pattern. OK? So when N is equal to 3, you have a family of two adult and one child.

[LAUGHTER]

Right? So basically, you have two big peak. And between them, there's a small peak. OK? That's actually how I remember this pattern. And I think it's pretty nice, right? So you can have N equal to 4. It's a bigger family. You have two adults. The adults are slimmer, OK? All right?

[LAUGHTER]

Because they have a lot of work to do. Then they have two child. All right? N equal to 5, how many children do we have?

STUDENT: We have three.

YEN-JIE LEE: Three. Therefore, the adults are really frustrated. So they are even slimmer in a happy way, making it positive. And N equal to 6, woo. Oh my god, I have four children in the family. All right? So there are two things which we learned from here.

The first one is that the number of big peak, which I would call it principal maxima, the number of principal maxima is actually pretty similar as a function of delta. But the number of secondary maxima increase as a function of N value. N value is actually telling you how many slits you have in the experiment.

And also, you can see that the delta is actually becoming-- the first minima, the delta value is actually decreasing as a function of N value. Right? So the parents are getting slimmer. All right? So therefore, you can see that if I would like to have a radar which is actually pointing to a very specific direction, what essentially the choice of N value which we will need? Infinity or a very large number. OK?

For sure in your life, we cannot do infinity. But now, we have found a way to actually design our radar since sine theta is actually proportional to delta. Therefore, what we actually really need to do is to really maximize the number of slits we have so that actually we can create a radar which would really point toward the direction of the enemy, which is shown there, invading the earth. OK.

[LAUGHTER]

And we can actually detect it. OK. So we will take a five minute break before we actually go to the last part of the course, which is the connection to quantum mechanics. So we come back at 35.

[SIDE CONVERSATIONS]

[SIDE CONVERSATIONS]

YEN-JIE LEE: OK so welcome come back from the break. So before we move to the connection to quantum mechanics, I would like to talk some more about what we have learned from the design of the radar. OK? So this essentially what we actually get. The position of the minima that required the phase difference delta is actually equal to 2 pi divided by N value, because it was this delta value. The N vectors is going to cancel each other. And you are going to form something like a circle if you choose delta equal to 2 pi divided by capital N. OK?

And don't forget why this is actually delta. The delta is actually d sine theta divided by lambda. Right? OK? And times 2 pi. OK? Right? So therefore, you can see that the sine theta is actually proportional to lambda divided by N times d. OK? And in this case, you can see that if you increase N value, the resolution or the width of the central principal maxima is going to be decreasing as a function, though, N value you're putting.

So in short, how do I actually design a high-resolution radar? What I really need is to have lambda to be small. OK? So that means I need to use high-frequency electromagnetic wave. I can maximize the N value. I can actually make d very large. That means I'm going to have a very large radar design. Right? Then I can have a very good resolution. OK.

So we are almost done with radar. But there's a problem. The problem is that if you look at this, if this is actually the position of the principal minima, you can see that is always pointing to the center of the radar where the delta is equal to 0. OK? And then that means I can only scan in one direction. There is a reason why those radar are called phased radar. That is because now I can actually change the relative phase of all those point source emitted from the radar so that I can shift the direction of the central principal maxima. OK?

So what is actually done here is like this. So basically, I can have introduced before emitting the electromagnetic wave, I can introduce a zero additional phase difference. And for the second one, I introduce additional phase difference of phi. OK? And for the third one, I introduce additional phase difference between the third slit-- or say the third emitter and the first emitter by 2 delta. And for N's emitter, I introduce a phase difference of N minus 1 phi. OK?

If I add this phase difference into the setup, what I'm going to get is like this. So basically, delta will become 2 pi divided by lambda d sine theta minus phi angle. All right? And this phi is actually the artificial eddy phase difference between those source. OK? And that means I will require-- and this will be equal to 2 pi divided by N value, such as you have completely destructive interference. OK?

I can now make this phi to be time-dependent. For example, it's increasing as a function of time, phi times t, right? Then what is going to happen is that as a function of time, I'm going to change the sine theta value so that I can get a complete cancellation, 2 pi over N. Right?

So effectively, I'm changing the angle of the central principal maxima by introducing additional artificial phase difference between all those point source. OK? And this is actually the way we can actually rotate the place we are scanning up and down and get a very nice result to detect the enemy. OK? Any questions? No?

OK. So now, I'm going to move on and discuss a very interesting experiment. So this is very exciting experiment content, billiard balls and the two slits. OK? And we will wonder, then, what is going to happen when those balls especially pass through the slit. Can anybody actually tell me what she is going to happen? And what will be the statistics, or say, the count, which I am going to go on to get in the receiver later? Anybody can actually tell me? If I actually shoot a lot of balls through this slit-- don't be shy, right? It's easy. No? Nobody wants--

STUDENT: They make [INAUDIBLE]

YEN-JIE LEE: Yeah, that's right. Right? Doesn't surprise nobody, right? [LAUGHS] Yeah, too afraid of answering questions. OK, you can see that they make two path, right? No? Right? OK. Very good. So now, this is the exciting part.

Now, instead of shooting billiard balls, what I'm going to do is to shoot electrons. So I can actually prepare an electron source and heat it up, such that it start to emit electrons. And I have two slits and have them pass through this slits. And I have a screen, which actually have an electron detector to count the number of electron which I am going to get on the screen.

The reason why I call it single electron source is because each time I control my experiment such that it only emit one electron every time. OK? The question I'm trying to ask is, will I see some pattern, which is actually light like billiard balls, and they form two piles in a pack? That's actually option number one. Or I'm going to see really something crazy? It's the electron is going to be interfere-- it's going through the interference with itself. And that essentially option number two. OK?

The lure of 8.03 is that everybody had to choose one. OK? So how many of you think what is going to happen is number one? Come on. I have only one electron each time. Nobody think so? Wow. Maybe all of you are wrong. [LAUGHS] How about the second option?

STUDENT: [LAUGHS]

YEN-JIE LEE: Hey, some of you actually didn't raise your hand. Come on. Come on.

[LAUGHTER]

OK, everybody. Wow. What is actually happening to you brain?

[LAUGHTER]

My brain is not functional like this. OK. So I really hope that I can bring the experiment to here. But unfortunately, that's actually going to be difficult. OK? So what I'm going to do is that I'm going to show you the experimental result, this video. And we are going to see what is going to happen.

You see that there are dots popping out. What are those? Those are the detected electron one-by-one on screen. OK? So basically, you can see that the number of dots are increasing as a function of time. And I actually-- I mean, speeding up things a bit so that actually you can see the pattern quicker. OK. So you can see that there are more and more dots.

And each time, you can see that I only get one electron per image here. Right? So you can see now there are more and more and more and more, and accumulating more data, like what we actually done in The Large Hadron Collider. We wait there, collect more data. And we are speeding things up. And you can see that, wow, something's actually developing. What is that? Can you see it? Now, you are speeding up like 1,000 times faster. You can see what pattern?

STUDENT: Interference pattern.

YEN-JIE LEE: Interference pattern. What is going on? You are not surprised?

STUDENT: No.

YEN-JIE LEE: Oh my god. What is going on?

[LAUGHTER]

I'm so surprised. Look at this. So I have emission of one electron each time. And that is actually the four snapshot which I took-- which actually this experiment, Hitachi Group actually did this experiment. You can actually click on this link to the more detail. And they took four snapshots of the experiment. And you can see that in the beginning, you can see clearly each time you only get one electron out of the source. OK?

But as a function of time, you're accumulating more and more. And you see that clearly, there's a pattern forming, which, is actually consistent with what we see in this calculation. OK? So I think that's actually truly amazing.

And what does that mean? That means the electron is playing with itself. It's interfering with itself. Right? That's really strange. What is going to happen? What is going on?

So one single electron pass through both slit, which is actually the option you choose. Surprise me. And then they interfere like waves. And they produce the pattern which we see on the screen. That is actually really crazy to me.

What is actually even more crazy is this situation. So now, if I make measurement in front of the slit, OK, so now, I puts on a little device. When the electron pass through one of the slit, I say, send me a signal. OK? So now, I can clearly know that which slit the electron is actually going through in the experiment. OK?

And the crazy thing is that if I do that, then it becomes two piles. OK? Of course, maybe there are some diffraction pattern. But it really changes the pattern of the experimental result. And that is actually really very strange. And we are going to talk about that briefly in the next lecture.

So before the end, I'm going to show you an additional demonstration which motivate the discussion what we are going to have in the next lecture. So now, I can actually turn off the light again and also hide the image. OK. I hope I can find the pattern. [LAUGHS] All right.

So here, I have two laser. So I'm going to turn up the first laser. And this laser is going to pass through a two slit-- a two really nearby slit and form an interference pattern. As you can see on the wall-- I hope you can see, I don't know if you can see clearly-- that you can see there are many, many dots, nearby dots, which actually shows you the position of the principal maximas, right, because are actually two slit experiment.

Therefore, how many children do we have in the family? Zero, right? Because they are-- OK, they just got married, maybe. [LAUGHS] All right. So therefore, you will see only adults. And that is actually the principal maximas. You can see many, many nearby dots. They are almost equally bright. OK?

But there's something happening to this pattern as well. And you can see that-- wait, wait, wait a second. In the calculation we get the principle maxima to have the same height, right? That means you are going to get exactly those same intensity for all the maximas. But you don't see that here.

You can see that if you move away from the center too much, the intensity is decreasing. You see at the edge? It actually even goes to zero. Right? What is actually happening? Something clearly is actually missing in our calculation. And that missing part is actually diffraction, which we will talk about that in the next lecture.

So if you compare this pattern to the second demo, you can see in the right hand side setup, which I have here, which I should give you a projection on the wall, which is actually lower part of the demo, you can see that this laser actually pass through a single slit. But this slit is actually pretty wide. OK? And you can see that indeed, you see the laser coming out, but essentially, not a single spot. And it has some kind of pattern, which is actually popping out there. And this is also related to interference between infinite number of source. OK?

And you can see that the pattern seems to really pretty similar to the pattern we see in the upper demo, except that upper demo have individual similar structure, which is the principal maxima from the two slit interference. And we are going to solve the mystery in the lecture next time. OK. So thank you very much. And if you have any questions related to the lecture today, I will be here to answer your questions.

So this is a demo which we would like to show you, Single Slit and the Double Slit Interference Pattern. OK? So the first scene is the setup. So we have a laser beam, which is actually passing through this either single slit or double slit experiment. And then the laser beam will be going through this and interfere and show interesting pattern on the screen. And there are two setup. The left-hand side one is two slit interference experiment. And right-hand side is a single slit diffraction experiment.

So you can see left-hand side one, I already turned it on. Laser beam passed through two slits. And they form complicated pattern on the screen. And you can see there are two kinds of structure here. The first one is the very fine structure, which you can see that it's like some row of dots in the center of the pattern. And there are larger scale pattern as well, which you can see that the overall intensity of all those little dots are also variating as a function of distance with respect to the center.

So during the lecture, we were wondering what actually cause this kind of pattern. And the answer is that this is actually coming from the effect of single slit interference. The reason why we have this pattern is because the two slit is actually not infinitely narrow in my setup. Therefore, within a single slit, there is already a interference pattern coming out of it. Therefore, the compound effect, results in a very complicated structure we see on the screen.

So to demonstrate this effect, now, I'm going to turn on the right-hand side setup. In the right-hand side setup, I am going to have the laser beam, which you see emitting from here, pass through a single slit. I actually set it up so that they have the same width between the single slit experiment and double slit experiment. And then you can see after I turn it on, you can see that now, we have two sets of pattern.

The lower set is actually coming from a single slit interference experiment. And you can see very nicely that first of all, it has a similar pattern, like what we see in the double slit experiment. Secondly, you can see that basically, we carefully tune these two experiments so that the distance between the slit and the screen is roughly the same.

Finally, we also set it up, as I've mentioned before, such that the width of the individual slit in the double and the single slit experiment are the same. And you can see that with single slit experiment, we also see a very similar pattern that you have a central maxima. You have a high-intensity light going toward the center of the pattern. And the intensity actually decrease dramatically really quickly as a function of distance.

And also, you can see that the pattern actually matches with what you see in the double slit experiment very well. And that is actually pretty remarkable. And from these two experiment, we understand why we have also a complicated structure in the double slit experiment, not just like many, many little maximas, many, many little dots. But also, you have this overall modulation in the light intensity. And that is actually mainly coming from the single slit diffraction pattern.