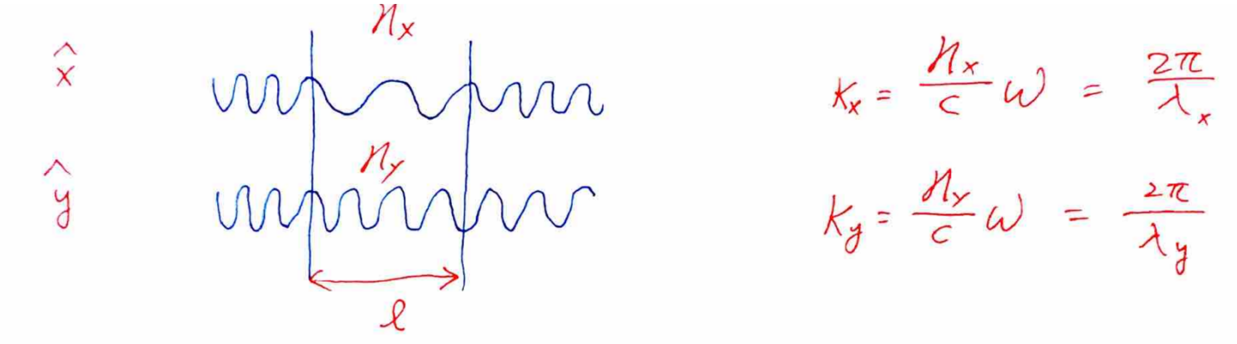
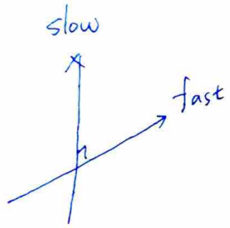
8.03 Lecture 18

Waveplate: use material which the index of reflection is different for different orientations of light passing through it!



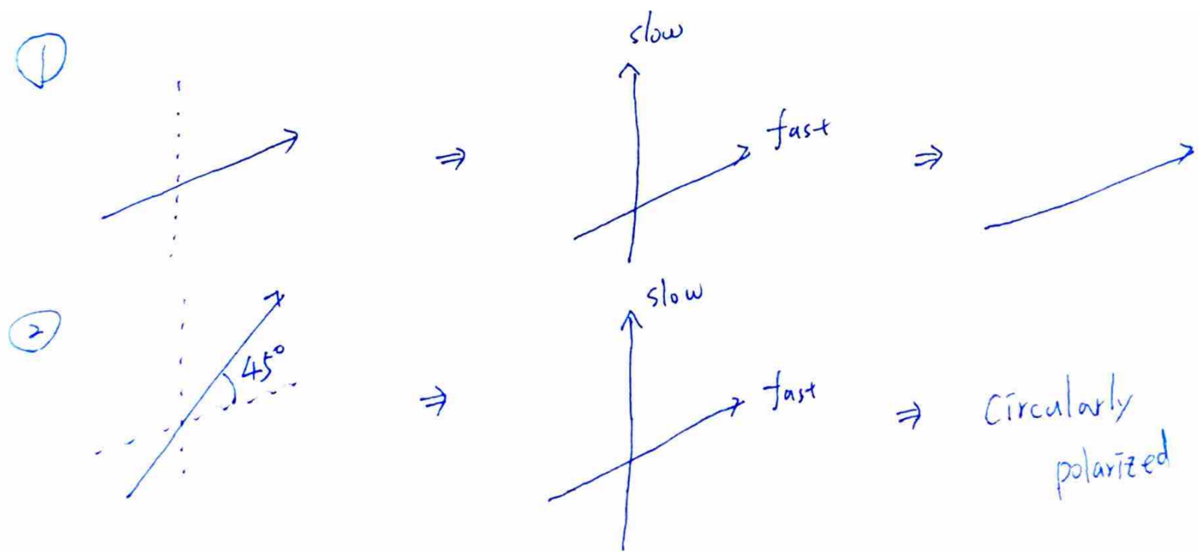
$$\Delta\phi = \frac{2\pi l}{\lambda_x} - \frac{2\pi l}{\lambda_y} = \frac{n_x - n_y}{c}\omega l$$

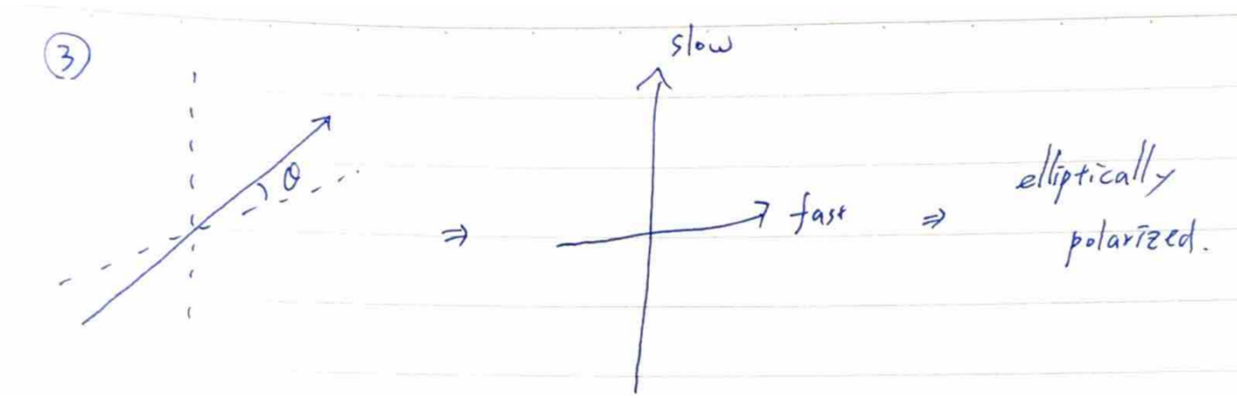
Quarter-waveplate: $\Delta\phi$ is designed to be $\frac{\pi}{2}$



*Axis with smaller phase → fast axis

*Axis with larger phase → slow axis





Matrix: $Q_0 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

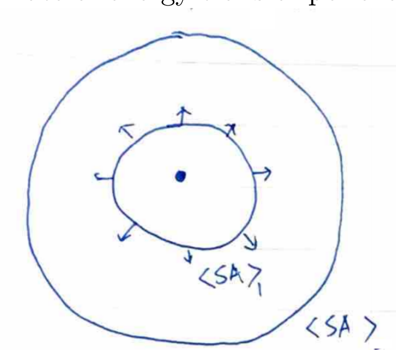
In general:

$$\begin{pmatrix} \cos^2\theta + i\sin^2\theta & \cos\theta\sin\theta - i\sin\theta\cos\theta \\ \cos\theta\sin\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$

Where θ is the direction of the fast axis with respect to the x axis.
(Editor's note: see video lecture for a demonstration.)

How do we produce EM waves?! Radiation from a point source.

In vacuum, EM wave neither loses nor gains energy. Recall the Poynting vector: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$
"rate of energy transfer per area"



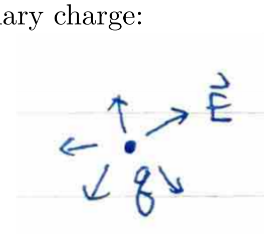
$$\langle S \cdot A \rangle_1 = \langle S \cdot A \rangle_2 = \text{power}$$

$$\langle S \rangle \propto 1/A \propto 1/r^2$$

$$\Rightarrow \langle \vec{E} \rangle, \langle \vec{B} \rangle \propto 1/r$$

Question: How do I produced radiation?

i Stationary charge:



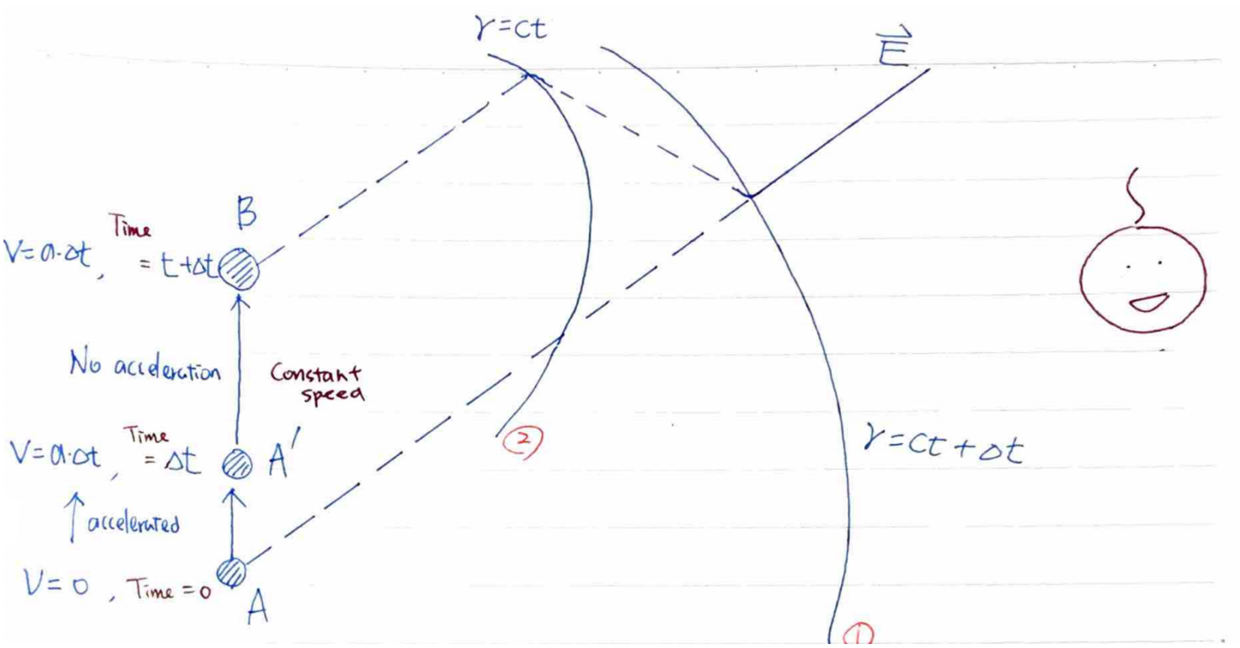
$$\left.\begin{array}{l} \vec{E} = \dfrac{q}{4\pi\epsilon_0 r^2} \propto \dfrac{1}{r^2} \\ \vec{B}_0 = 0 \end{array}\right\} \vec{S} = 0$$

ii Charge at constant speed u:

$$\beta = \frac{u}{c}$$
$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}$$
$$\vec{B} = \frac{\vec{u}\times\vec{E}}{c^2} \propto \frac{1}{r^2}$$
$$\Rightarrow |\vec{E}| \propto \frac{1}{r^2} \quad , \quad |\vec{B}| \propto \frac{1}{r^2}$$

$$\frac{1}{\mu_0}\vec{E}\times\vec{B} = \vec{S} \propto \frac{1}{r^4} \Rightarrow \text{Does not radiate}$$

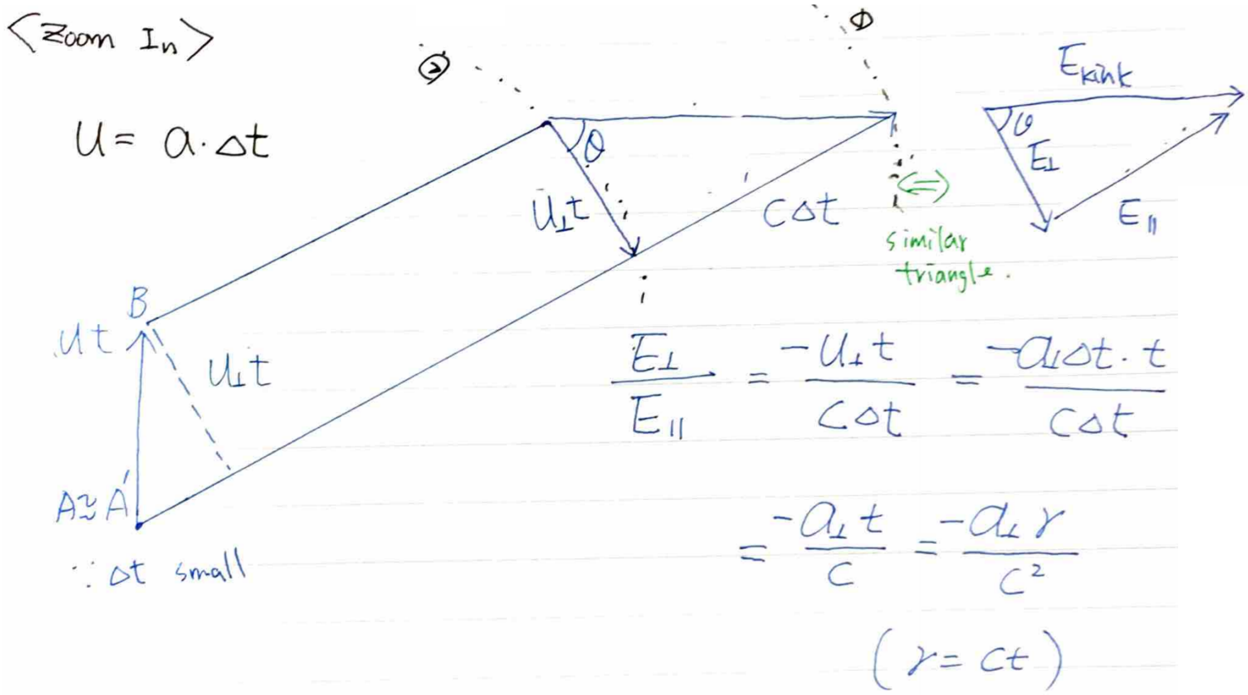
(Or we can use a simpler argument: boost to the rest frame of the charge)
Therefore we need to accelerate the charge to produce radiation. (Proof can be found in Georgi 355-360). Or the following geometrical argument. Goal: to create a "kink" in the electric field: Accelerated Charge!
Consider a charge, accelerated between $t = 0$ to $t = \Delta t$. a is small and Δt is small.



It takes time for information to propagate (at the speed of light).

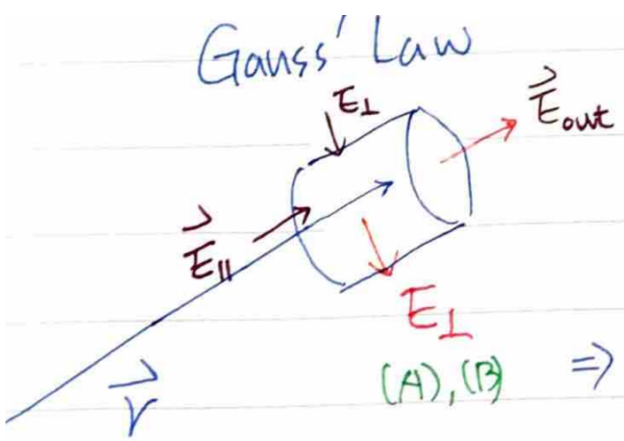
(1) Surface: information that the charge accelerated has only just reached this sphere
(2) Surface: information that the charge moving with constant velocity has reached this sphere
Q:What will the "observer" see at $t = t + \Delta t$? A: A stationary charge.
Therefore outside (1) the electric field is like the charge has never moved (where the observer lives). Inside (2) the electric field is in the $\hat{r}$ direction. Between (1) and (2) the field must be continuous because there is no source between them. Since $u \equiv a \cdot \Delta t$ is $\ll c$ (where u is the velocity of the

charge after acceleration) then the field lines from A to B are approximately parallel. We have managed to create a "kink"!



$$\Rightarrow E_\perp = \frac{-a_\perp r}{c^2} E_\parallel$$

What is $E_\parallel$? Use Gauss' Law:



$$E_\parallel = E_{Out} = \frac{q}{4\pi\epsilon_0 r^2} = \text{Electric field outside}$$

$$E_\perp = \frac{-q a_\perp}{4\pi\epsilon_0 r^2 c^2}$$

This is very important! $E_\perp$ at position $\vec{r}$ is due to acceleration which occurred at a retarded time:

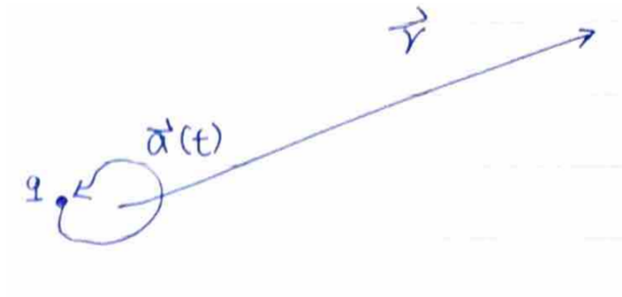
$$t' = t - r/c$$

$$\Rightarrow \vec{E}_{Rad}(\vec{r},t) = \frac{-q\vec{a}_\perp(t-r/c)}{4\pi\epsilon_0 c^2 r}$$

$$\Rightarrow \vec{B}_{Rad} \propto \frac{1}{r}$$

$$\Rightarrow \vec{S}_{Rad} \propto \vec{E}_{Rad} \times \vec{B}_{Rad} \propto \frac{1}{r^2}$$

We are sending energy to the edge of the universe!!



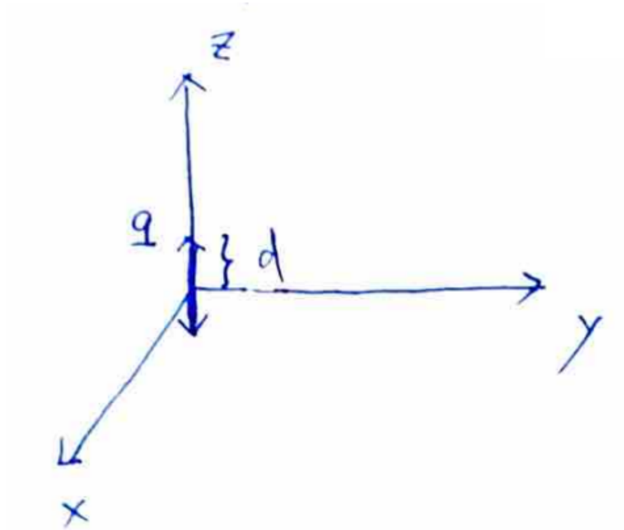
$\vec{r} \gg$ scale of $\vec{a}(t)$ such that the static contributions die out.

$$\vec{E}_{Rad}(\vec{r},t) = \frac{-q\vec{a}_\perp(t-r/c)}{4\pi\epsilon_0 c^2 r}$$
$$\vec{B}_{Rad}(\vec{r},t) = \frac{1}{c}\hat{r} \times \vec{E}_{Rad}(\vec{r},t)$$
$$\vec{S}_{Rad}(\vec{r},t) = \frac{1}{\mu_0}\vec{E}_{Rad} \times \vec{B}_{Rad}$$

$$\vec{a}_\perp = \vec{a} - \vec{a}\cdot\hat{r}\;\hat{r}, \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

1. Get $\vec{a}$
2. define $\vec{r}$, get $\vec{a}_\perp$ $\quad \vec{a}_\perp = \vec{a} - \vec{a}\cdot\hat{r}\;\hat{r}$
3. $\vec{E}_{Rad}$
4. $\vec{B}_{Rad} = \frac{1}{c}\,\hat{r} \times \vec{E}_{Rad}$
5. $\vec{S}_{Rad} = \frac{1}{\mu_0}\vec{E}_{Rad} \times \vec{B}_{Rad}$
6. Total power: $P(t) = \iiint \vec{S}_{Rad}(\vec{r},t)\cdot dA\hat{n} = \frac{q^2|a(t-r/c)|^2}{4\pi\epsilon_0 c^3}$

Example: harmonically oscillating charge:



where $x = \hat{z} d \cos \omega t$ and $R \gg d$

(1) At a distance R away from the charge in the $\hat{z}$:

$$\vec{a}(t) = \ddot{\vec{x}}(t) = -\hat{z} d\omega^2 \cos \omega t$$

$$\vec{E}_{Rad}(\vec{r}, t) - = \frac{-q\vec{a}_\perp(t - r/c)}{4\pi\epsilon_0 c^2 r}$$

$$\vec{a}_\perp = \vec{a} - \vec{a} \cdot \hat{r}\hat{r} \ \text{ in this case } \vec{a} \parallel \vec{z}$$

$$\Rightarrow \vec{a}_\perp = 0$$

$\Rightarrow$ No radiation!

(2) How about $R\hat{y}$?

$$\vec{a}_\perp = \vec{a} - \vec{a} \cdot \hat{y}\hat{y} = -\hat{z} d\omega^2 \cos \omega t$$

$$\vec{E}_{Rad}(t) = \frac{+qd\omega^2 \cos(\omega(t - R/c))}{4\pi\epsilon_0 c^2 R}\hat{z}$$

$$\vec{B}_{Rad}(t) = \frac{1}{c}\hat{y} \times \vec{E}_{Rad}(t) = \frac{qd\omega^2 \cos(\omega(t - R/c))}{4\pi\epsilon_0 c^3 R}\hat{x}$$

We get harmonic waves with amplitude decreasing versus R (3) How about at $R\left(\frac{1}{2}\hat{y} + \frac{\sqrt{3}}{2}\hat{z}\right)$?

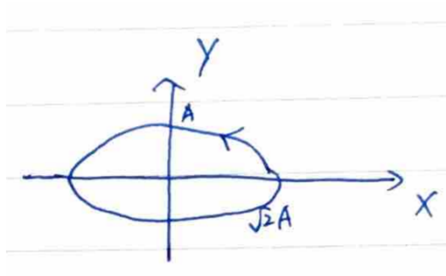
(30° angle with respect to the z-axis in the $y-z$ plane)

$$\begin{aligned}\vec{a}_\perp(t) &= \vec{a} - (\vec{a}\cdot\hat{r})\hat{r} \\ &= -\omega^2 d\cos(\omega t)\left(\hat{z} - \frac{\sqrt{3}}{2}\left(\frac{1}{2}\hat{y} + \frac{\sqrt{3}}{2}\hat{z}\right)\right) \\ &= -\omega^2 d\cos\omega t\left(\frac{1}{4}\hat{z} - \frac{\sqrt{3}}{4}\hat{y}\right)\end{aligned}$$

$$\vec{E}_{Rad} = \frac{q\omega^2 d}{8\pi\epsilon_0 c^3 R}\cos(\omega(t-R/c))\left(\frac{1}{2}\hat{z} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

Example 2: A particle with charge q is moving on an elliptical orbit

$$\begin{aligned}x(t) &= \sqrt{2}A\cos(\omega t) \\ y(t) &= A\sin(\omega t)\end{aligned}$$



What are the polarizations of the electric field seen by distant observers on the positive x, y, z axes? First calculate $\vec{a}(t)$

$$\vec{a}(t) = -\sqrt{2}A\omega^2\cos(\omega t)\hat{x} - A\omega^2\sin(\omega t)\hat{y}$$

(1) Observer $R\hat{x}$

$$\begin{aligned}\vec{a}_\perp &= -A\omega^2\sin\omega t\hat{y} \\ \vec{E}_{Rad} &= \frac{q\omega^2 A}{4\pi\epsilon_0 c^3 R}\sin(\omega(t-R/c)) \quad \text{Linearly polarized}\end{aligned}$$

(2) $\hat{y}$: similarly, also linearly polarized
(3) $\hat{z}$: elliptically polarized

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