

Massachusetts Institute of Technology

Physics 8.03SC Fall 2016

Homework 1

Problems

Problem 1.1 (20 pts)

For the mass and spring discussed during the lecture (Howard Georgei Eq.(1.1)-(1.8)), suppose that the system is hung vertically in the earth's gravitational field, with the top of the spring held fixed. Show that the frequency for vertical oscillations is given by Eq.(1.5). Explain why gravity has no effect on the angular frequency.

Problem 1.2 (20 pts)

In each of the following three cases, the motion of a particle is specified by a complex representation of its displacement, velocity, or acceleration. For each case find a real representation for all three quantities – displacement, velocity, and acceleration – in the form:

$$\eta(t) = c \cos \omega t + d \sin \omega t$$

where c and d are real, time independent quantities. A , B , C , and τ are real quantities. In case you need to integrate assume that the integration constants are equal zero.

a. $X(t) = Ae^{-i(\omega t - \pi/2)}$

b. $\dot{X}(t) = Bi\omega\tau e^{-i(\omega t + \pi/6)}$

c. $\ddot{X}(t) = C \frac{1}{1+i\omega\tau} e^{-i\omega t}$

In each of the following cases the motion of a particle is specified by a real representation of its displacement, velocity, or acceleration. For each case find the complex representation for all three quantities – displacement, velocity, and acceleration – in the form

$$Q(t) = Qe^{-i(\omega t + \phi)}$$

where Q and ϕ are real, time independent quantities. α , β , γ and δ are real and positive quantities.

d. $x(t) = \alpha \cos \omega t - \beta \sin \omega t$

e. $\dot{x}(t) = -\gamma \sin(\omega t + \pi/3)$

f. $\ddot{x}(t) = \delta \cos(\omega t + \pi/6)$

Problem 1.3 (20 pts)

A block of mass M slides without friction between two springs of spring constants K and $2K$ as shown in Figure 1. The system is constrained to move only along the axis of the springs.

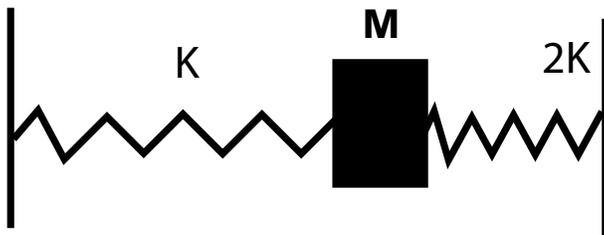


Figure 1: Oscillating mass

- Calculate the angular frequency of oscillations.
- If the velocity of the block when it is at the equilibrium position is v , calculate the amplitude of oscillations.
- Write an expression for the position of the mass as a real function, $x(t)$ (with the initial condition from (b)).
- Write an expression for the position of the mass as a complex function, $z(t)$, in the irreducible form (with the initial condition from (b)).

Problem 1.4 (20 pts)

A particle of mass m moves on the x axis with potential energy

$$V(x) = \frac{E_0}{a^4} x^4 + 4ax^3 - 8a^2x^2$$

- Find the positions at which the particle is in stable equilibrium.
- Find the angular frequency of small oscillations about each stable equilibrium position.
- What do you mean by small oscillations? Be quantitative and give a separate answer for each point of stable equilibrium.

Problem 1.5 (20 pts)

Consider a simple pendulum consisting of a point-like mass m attached to a massless string of length L hanging from a fixed support and constrained to move in a vertical plane (see Figure 2). Assume gravitational acceleration to be g .

- Parametrize the motion of the pendulum in terms of the angle θ , its deviation from the vertical. Find the exact equation of motion ($\vec{\tau} = I\vec{\alpha}$) for the pendulum as a function of θ .

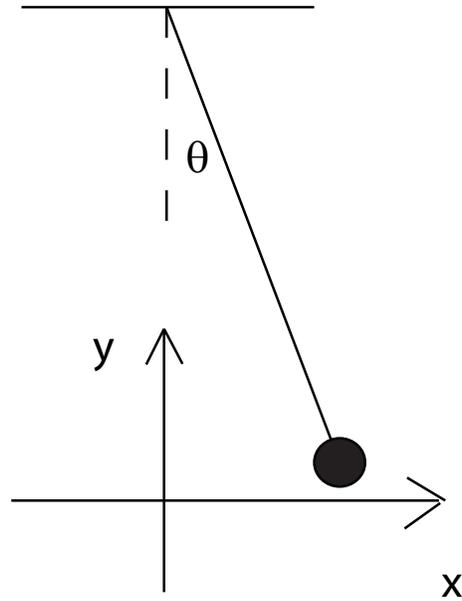


Figure 2: Pendulum

- b. Assume that the angle θ is small and find the approximate simple harmonic equation of motion.
- c. Justify your approximations. Find the range of θ that the pendulum can be considered a SHM. What is the period of oscillations of this SHM?
- d. Calculate the exact potential energy of the pendulum as a function of θ . Then, show that the Taylor expansion leads to the same result as in part (b).
- e. Parametrize the motion of the pendulum in terms of the cartesian coordinate x in the coordinate system with origin at the pendulum equilibrium position and x -axis horizontal in the plane of pendulum. Find the exact equation of motion ($\vec{F} = m\vec{a}$) of the pendulum in terms of x .
- f. Assume that x is small and find the approximate simple harmonic equation of motion.
- g. Justify your approximations. Find the range of x such that the pendulum can be considered a SHM.

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8.03SC Physics III: Vibrations and Waves
Fall 2016

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