PROFESSOR: Let me begin by introducing the subject. The subject is resonances. And we have seen, actually, a little bit of this in the resonant transmission of the Ramsauer-Townsend effect. Because of a resonance phenomenon within the square well obstacle, somehow, for some particular frequencies, for some particular energies, the particles were able to zoom by without experiencing any reflection, whatsoever.

So let's begin the subject of resonances by asking a question. If you have the usual potential, the short range potential, which means, that for some distance, R, greater than 0, the potential is 0. Here we put a barrier, and over there could be anything, some potential. We've computed some-- this concept of time delay, there's a formula for the time delay. In fact, it was given by 2 h bar d delta dE, the time delay, 2 h bar d delta dE.

And we discussed that this time delay can be positive or it can be negative. If it's positive, it really means a time delay. You send in a wave packet. And it takes time to come back, more time than it would have taken if there had been no potential.

You see, the time delay, you have a packet coming in from time minus infinity. And then it bounces back a time equal infinity. But nevertheless, you compare that with a situation in which there's no potential. And you see that there is some time delay. If you time the wave packet to reach at time equals 0, here, it will not reach back to where you were by time-- by whatever time-- suppose you have the wave packet here at t equal minus 10, then it goes here, and it delays, and at t equal 10, the packet has not reached, there is a time delay, a positive time delay.

A negative time delay is the opposite. The packet arrives a little earlier. And the question I want to ask you, if you have a negative time delay, can it be arbitrarily large. Well, if you send in a wave packet, it may find an infinite wall here, and then may bounce, and then yes, it comes back earlier than you expected, because the free packet would have gone here and back.

But you wouldn't expect it to be able to come earlier than if there was an infinite wall here, because there is no infinite wall here, nor an infinite wall here. So it's just not going to bounce before it reaches here. The best it can do is bounce when it reaches here. So you should not expect, and this, sometimes, will [INAUDIBLE], there is nothing that can make it bounce until you reach here. So you cannot expect that the time advance is larger as if it would have bounced before reaching the obstacle, whatsoever.

So this is important. We cannot have a negative time delay that this infinitely large. So, in fact, the time delay as, we're right in here, should be greater than the total travel distance that you may save. If you bounce here, you would save 2R over v. And you must be greater than that negative number, which is the total travel time that it would take to go back and forth, here.

So we can do a little arithmetic, here. This is equal to 2 h bar d delta dk, and here, dE dk. This is still greater than or equal than minus 2R over v. And I want to put a sim, because our argument is not completely rigorous as to what's happening when it reaches here. It seems very plausible classically, but there's a bit of a correction if you do it exactly. So it's not an exact inequality we're deriving.

And what is the E dk is h bar times the velocity. Remember, dE dk, you are differentiating h squared k squared over 2m. And you get h bar times hk over m. So therefore, this is h bar and the velocity. And the h bars cancel. The velocities cancel. Between these two sides, the 2s cancel. And you'll get that d delta dk must be greater than or equal, approximately, to R.

And that's sometimes called Wigner's condition on scattering. And it basically is the idea that the time delay, the time advance cannot be too large. OK, so now we can ask the second question. How about the time delay, a true time delay, can it be very large? Can it be arbitrarily large?

Suppose we have a barrier of this form. And now you send a particle with a little bit higher energy here. Now, this particle is going to have very little kinetic energy. So it's going to travel quite slowly here, and go back. And this time, it's going to delay quite a bit, probably. But the problem is, if you create-- there's nothing very peculiar about this, if you go a little lower, than you're advanced, and then suddenly, it gets delayed. It's not that evident, but the phenomenon of resonance is precisely what we get when we, sort of, trap the particle. And then we make it be, as far as it seems, arbitrarily large, if you design a well properly.

But the thing that we have to design, the example of what we're going to design, is different from all the things I've drawn so far. It's the following way, this is just an example. I have this zero line of the energy. This is v of x. This is x. And then I put an attractive potential here. And here is minus v0. And then I put a barrier here with a v1.

So what I'm going to aim at is, you see, if v1 will be extremely large, there will be-- well, if v0 is extremely large, then begin there would be bound states here, but these are not scattering states. On the other hand, if v1 will also be infinite, you would have bound states here, but they could not escape. So certainly, if I combine these two, I put a v0 and maybe a larger v1, I can almost create bound states here.

But they're not really bound states, because they can leak out and produce scattering states. But these are going to be resonances. This part and this, this being a attractive, trying to keep the particle in, and this being a barrier, can combine to produce a state that gets trapped here, and stays a very long, time, will have a very long time delay.

And that's the phenomenon of resonances. We need to trap that particle, somehow. And we're going to see now the details of how this works, and what the properties are. Now, it's very interesting that actually, these resonances occur at some particular energies. And they have different properties.

But we can identify energies of resonances. And these are not bound states. They're just resonances. They eventually escape. And they're not normalizable, really, but in some ways they behave as bound states for awhile. They stay there for a while and do nice things.

So let's set this off. Now we're going to spare you a little bit of these calculations, because the important thing is that you know how to set it up, and if you get an answer, you know how to plot it, how to get the units out, how to try to understand it. So that's what we're going to do.

I'm going to put an energy here, an energy, E. And I'm going to receive E to be less than v1 and greater than 0. I don't expect true resonances beyond, because the particle just bounces out. It doesn't get trapped. The phenomenon of resonance is a little more intricate than just having a long time delay. There's more that has to happen.

Another thing that will happen, is if the particle spends a lot of time here, you would find, in this spirit of resonance, that the amplitude of the wave function here is going to be very big. So you will scan the energy and the amplitude. It will be normal, normal, normal. And suddenly for some energy it becomes very big. And we're going to do that.

The way I'm going to develop that, we're going to calculate this, plot these things. And then we are going to ask whether there is a mathematical condition that picks resonances. Well, how do I, if I want to explain to somebody in 30 seconds where are the resonances, how do you calculate them, you cannot tell that somebody, OK, calculate it for all energies, do all the plots, and see some peak in some thing, and this is a resonance. This is what we're going to do to begin with, but then we'll get more sophisticated.

So let's put k in. So let's call this k prime, the wave number in this area. Kappa here, because it's a forbidden region, and k over here, as usual. So k squared is 2mE over h bar squared. K prime squared is equal to 2m, the total kinetic energy is E plus v0, over h squared. And kappa squared is again, similar formula, but this time is the energy differential between v1 and E, so 2m v1 minus E over h squared.

All of these three numbers are positive. And they are the relevant constants to write wave functions. So we have to write a wave function. And I'm going to write a wave function because it takes a little tinkering to do it in an efficient way.

There is one that you don't have to think, you just have to remember. It's the one outside. It's the universal formula, e to the i delta sine kx plus delta is valid for for x greater than R. This one we derived at the beginning of our analysis of scattering.

How about the other region. Oops, I should have put letters here. These are a and 2a they are positions. And therefore, it's not R in here. Well, it's R, it's the range of the potential, but here is 2a.

How about the other one? In this region, it's kind of simple again. The wave function has to vanish here, has to be sines or cosines of k prime. So it has to be a sine function of k prime. And since we don't put an extra constant in here, we kind of put an extra constant in here, there must be a constant here, A sine of k prime x. And that must be for x between 0 and a.

We used k prime, the wave from over there. And there is A. And what we were saying about resonances, is that, well, A may depend on k. And when you have a resonance, A is going to [INAUDIBLE], presumably because the particle spends a long time inside the well.

And now I have to write this one in here. And this is the one that, you can do it, do a little bit more work, or do it kind of, efficiently. In that region we have exponentials, like we have e to the kappa x and e to the minus kappa x. Or I may want to have sinh of kappa x and cosh of kappa x to write my solutions.

But I actually don't want either of them too much, because I would like to write an answer that almost imposes continuity in a nice way. So I could use sinh of kappa x minus a and cosh of kappa x minus a. These are all solutions. You can choose whichever pair you want.

So for example, if I want to implement continuity with this thing, this wave function, I want to write something that I don't have to write another equation for continuity. So I will write A sine of k prime a-- so far, this wave function, if x equal a, coincides with this one. But this is no wave function yet, not with an x dependence, so I have to put more. But then, I know that cosh is 1 for x equals zero. So I put here a cosh kappa x minus a.

And now this is a solution that matches that one at x equals a. At x equal a, the cosh becomes 1 and matches. But this kind of need a complete solution. It's not general enough. So you have to put a B sinh of kappa x minus a.

And this won't ruin the matching, because at x equal a, that second term vanishes. So we're still matching well there. And Well matching here is non-trivial when I impose some conditions. So you still have to match derivatives and do a little bit of work but not too much work.