

PROFESSOR: We ask, is $\psi(x, 0)$ real? And I told you the answer is no. And how would I know that this is not real? Well, we can take the complex conjugate. And at the end of the day, this will boil down to some property of $\psi(k)$.

You see, you have an expression $\psi(x)$ in terms of $\psi(k)$. So it would not be surprising that the requirement that ψ is real means something about $\psi(k)$.

So let's just say, suppose ψ is given by that, then $\psi(x, 0)^*$, the complex conjugate would be $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(k)^* e^{-ikx} dk$. I conjugated everything in that equation for $\psi(x)$ and 0.

Now, you want to compare this with $\psi(x, 0)$ to see if it's real. Or let's consider what is the condition that this be real. So I want to simplify here a little more. So what I'm going to do is going to change variables, by changing k to $-k$.

If you prefer to go a little more slowly, you could say you're going to change k prime-- you're going to be a new k , called k prime, equals to $-k$. But it's possible to do it this way.

Now, there's going to be a couple of changes. Wherever you see k , you're now going to see $-k$ -- so $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(-k)$. And I'll just put this star here, not so many parentheses-- e^{-ikx} becomes e^{ikx} . And the dk will go to $-dk$, but the order of integration, that was from $-\infty$ to $+\infty$, would switch. So those two signs cancel.

So there's a sign from doing dk to $-dk$, and 1 from the limit of integration-- so at the end of the day, you have dk , and you still have this-- $-\infty$ to $+\infty$. And you can say, well, is this equal to-- or what is the condition of for ψ to be real? Well, is this equal to $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(k) e^{ikx} dk$ -- is that-- question mark-- is that equal to it?

That would mean that the $\psi(x, 0)$ is real, because this thing is just $\psi(x, 0)$. So this is a question mark-- this is a condition. So here you could say, exploring the reality condition-- condition-- when is a $\psi(x)$ real?

So what must be true is that these two terms must equal each other. So, in fact, this requires-- reality requires that $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(k) e^{ikx} dx$ is equal to 0.

I brought the two terms to one side. Both are of the same type-- they're integrated against an e to the ikx . And therefore, we can combine them, and that's what must be true in order for the function to be real. And now you can say, so what is it? What's the answer?

Well, this integral should vanish. Now, this integral should vanish-- it should vanish for all values of k . So actually, what you want to conclude is that this thing is identically 0.

AUDIENCE: Excuse me, shouldn't that be dk and not dx ?

PROFESSOR: Yes, thank you. Thanks very much.

So this property that this whole integral be equal to 0, you were tempted to conclude that it means that this thing is equal to 0. And that is correct-- that is a perfectly legal argument. And it basically-- if you want to express it more precisely, you could base it on the Fourier theorem, again. These two sets of equalities here are Fourier's theorem.

And look what this is saying-- this is saying that this quantity has a 0 Fourier transform.

Because how do you do the Fourier transform of a function of k ? You multiply by e to the ikx and integrate. And therefore, this function has a 0 Fourier transform.

So, but if a function has a 0 Fourier transform, the function must be 0. Because already this is 0, and the integral is 0-- 0. So this is absolutely rigorous. And therefore, you get the conclusion that ϕ of minus k star must be equal to ϕ of k , and that's the condition for reality.

So if a ϕ of k satisfies this property, that ψ of x will be real, and our ϕ of k doesn't satisfy this property, what do you see in this property? Basically, if you have ϕ that exists for some value of k , it should also exist for the value of minus k . And in fact, should be the complex conjugate of the other value.

But here, you have some ϕ s of k , and no ϕ s at minus k . So the ϕ that we wrote above doesn't satisfy this condition. And therefore ψ is not real, and it all makes sense.

OK, so basically, if you were plotting not the absolute value, but the real part and the imaginary parts of ψ , you would see some sort of funny waves. I think if you were plotting the real part, for example, you would see a wave like that. And if you were plotting the imaginary part, you'd presumably see some other wave like that. And the absolute value, it's much nicer and simpler.

