PROFESSOR: We have the hydrogen atom Hamiltonian. Hamiltonian. And that was given by the kinetic operator for the proton plus the kinetic operator for the electron plus the potential, which was a function of the distance between the proton and the electron.

And what we achieved last time was the introduction of two new pairs of canonical variables. We had the electron position momentum, that's a pair of canonical variables. The proton position and momentum, that's another pair of canonical variables. They commute each pair, the two operators commute to give IH bar, but the two pairs are independent.

So we search for another two pairs of variables, and we found another two pairs. One was the P and X associated with the center of mass motion, and then we had the small p and small x associated with the relative function. And these four variables were a function of the original four variables, the X and P of the electron and the x and p of the proton.

So we define these two pairs, and they were canonical pairs. This x with this p gave IH bar, this x with this p gave IH bar, these p's and x's commute with any combination of p's and x's over there.

But not only was that pretty good, it simplified the Hamiltonian. So at the end of the day, we had a Hamiltonian, which was as if the center of mass moves like a free particle plus a kinetic energy for the relative motion, with a mass called the renews mass, and the potential for the relative position. And here, the mass, capital M, was the sum of the two masses. And the relative mass was the product of the masses over the sum, which has the property that if one of the two masses is much bigger than the other, it gives you a mass mu proportional roughly equal to the lower mass.

So this is the hydrogen atom reformulated. And now, we want to write the Schrodinger equation and just see effectively how the central potential formulation of the relative motion arises. Although, it starts to be a little somewhat clearer, I think, that that's going to happen.

Another thing to notice of course, is that if you're already thinking of a Schrodinger equation, in which you will think of the momenta as the derivative operators, the center of mass momentum should be thought as the derivative operator with respect to the center of mass position. So think of the center of mass as three coordinates, and you differentiate with respect to them.

Similarly, for the relative momentum, we'll think of it as a gradient with respect to the relative position, because that's the canonical coordinate that goes along with it. That's how we should think of this operator as derivative.

And before, we didn't have to put those subscripts in [INAUDIBLE], because we always had just one coordinate to work with. But now, you have two coordinates. What you've learned here in doing this analysis was that we have a wave function that has coordinate dependence on both the electron and the proton.

So let's do the separation of variables that shows how to deal with this system. So you want to write the wave function for the whole system, so it depends on the center of mass and on the relative coordinates. Now, I don't put time, because we're discussing time independent Schrodinger equation, where the time you can put it later, if you wish, with the total energy into the minus IET.

So what we will consider is a simple solution that is of the product type. So there will be a wave function associated up factor, which is a wave function associated with center of mass, and the wave function associated to the relative motion. And we want to replace this into the Schrodinger equation into H psi equals E psi.

So let's see how it would go. Each term of the Hamiltonian is going to act on this product of functions that determines the whole wave function. The center of mass momentum, being derivative, respect to this x, will act just on the first term. So we'll have p squared over 2 m acting on psi c m of x. And the relative wave function in that term just goes for the ride. So that's the first term in the Hamiltonian.

For the second term in the Hamiltonian, we would have p squared over 2 mu. We could add them psi relative, and let's put even the potential here, v of x relative-- we don't put the relative on the x, but they'll just-- it's a small x psi relative of x. So we've looked at the second term and the third term. The third term is grouped with the second, because it uses the little x, not the big X. And then you have psi cm of x multiplicativly, it doesn't do anything to it.

All this is equal to e times the psi cm of x psi relative of little x. OK, that's the Schrodinger equation. And this should remind you, it's very similar to what you did months ago of having motion's in say, in two dimensions, and you wrote part of the wave function dependant on x, part of a wave function dependant on y, and you separated the Schrodinger equation.

So the next thing to do is to divide by the total wave function, by the product. So divide 5 by the total wave function. So what do you get? From the first term, you will get 1 over psi cm of capital X times this p squared over 2 m psi cm of capital X. And that's all what comes of the first term.

From the second, you get plus 1 over psi relative of x and this whole bracket squared over 2 mu psi relative plus V of x psi relative. And the whole thing being equal to E. The two wave functions are divided there.

So we have a situation where a number is the sum of two functions. Now, what is funny of course, is the argument you've heard several times. This first term depends just on the capital X-coordinates. The second term depends just on the lower x-coordinates, small x-coordinates. I didn't write them in some places, but here they are. And therefore, the only way these two things can always be true, is if the first term is a number. And we'll call it Ecm. The whole thing must be a number, we'll call it Ecm. And the second term should be another number, and I'll call it E relative. That's E relative.

So our conclusion is that if this first term is a number equal to Ecm we can multiply by psi cm and get p squared over to 2m psi cm of x is equal to Ecm times psi cm of x. Which is a time independent Schrodinger equation for a wave function psi cm that is moving freely.

The next equation is the one within brackets, which is p squared over 2 mu psi relative of x plus v of-- now I can put r, when I say that r is the magnitude of x psi relative of x equals e relative-- I should move this-- psi relative of x. And we said that r was this.

So the second equation comes from this term identified with relative. And the last equation is to say that the total energy is equal to Ecm plus e relative.

So the whole two body problem has been reduced to these three equations. This is what we aim to show. This is a gradient squared on this wave function, the central potential term, and the rest of the Schrodinger equation. So this is a Schrodinger equation for a particle in the central potential. That particle happens to be the relative distance between these two particles, but it obeys a central equation potential.

Here is the center of mass. Solutions of this is a plane wave, momentum plane waves, because this is like a free particle, and that's our intuition. The hydrogen atom can move like a free particle. That's an overall quantum system and then there's the relative degrees of freedom. The total energy of this system must be the sum of the two.

So that said for the system, we are allowed now, to consider the hydrogen atoms. So that's what we'll do next.