PROFESSOR: Superposition is very unusual and very interesting. Now we've said about superposition that in classical physics, when we talk about superposition we have electric fields, and you add the electric fields, and the total electric field is the sum of electric fields, and it's an electric field. And there's nothing strange about it. The nature of superposition in quantum mechanics is very strange. So nature of superposition-- I will illustrate it in a couple of different ways.

One way is with a device that we will get accustomed to. It it's called the Mach-Zehnder interferometer, which is a device with a beam splitter in here. You send in a beam of light-- input- beam splitter and then the light-- indeed half of it gets reflected, half of it gets transmitted. Then you put the mirror here-- mirror 1, you put the mirror 2 here, and this gets recombined into another beam splitter. And then if there would be just a light going in, here there would be two things going out. There's another one coming from the bottom. There will be two. There will be interference. So you put a detector D0 here and a detector E1 here to detect the light.

So that's the sketch of the Mach-Zehnder interferometer-- beam splitters and mirrors. Take a beam, spit the light, go down, up, and then recombine it and go into detectors. This was invented by these two people, independently, in the 1890s-- '91 to '92 apparently. And people did this with light-- beams of light before they realized they're photons.

And what happens with a beam of light-- it's interesting-- comes a beam of light. The beam splitter sends half of the light one way half of the light the other way. You already know with quantum mechanics that's going to be probabilistic some photons will go up maybe some photons will go down or something more strange can happen.

If you have a superposition, some photons may go both up and down. So that's what can happen in quantum mechanics. If you send the beam, classical physics, it divides half and half and then combines. And there's an interference effect here. And we will design this interferometer in such a way that sometimes we can produce an interference that everything goes to D0 or everything goes to D1 or we can produce suitable interferences that we can get fractions of the power going into D1 and D2-- D0 and D1.

So we can do it in different ways, but we should think of this as a single photon. Single photos going one at a time. You see, whatever light you put in here, experimentally, the same frequency goes out here. So what is interference? You might think, intuitively, that interference is one photon interfering with another one, but it can't be.

If two photos would interfere in a canceling, destructive interference, you will have a bunch of energy. It goes into nothing. It's impossible. If they would interfere constructively, you would add the electric fields and the amplitude would be four times as big because it's proportional to the square. But two photos are not going to go to four photons. It cannot conserve energy.

So first of all, when you get light interference, each photon is interfering with itself. It sounds crazy, but it's the only possibility. They cannot interfere with each other. You can send the photons one at a time and, therefore, each photon will have to be in both beams at the same time. And then, each photon as it goes along, there will be an interference effect, and the photon may end up here or end up there in a probabilistic way. So you have an example of superposition.

Superposition. A single photon state a single photon is equal to superposition of a photon in the upper beam and a photon in the lower beam. It's like two different states-- a little different from here, you had photons in two different polarizations states superposed. Here you have photons in two different beams-- a single photon is in both beams at the same time. And unless you have that, you cannot get a superposition and an interference that is consistent with experiment. So what does that mean for superpositions?

Well, it means something that we can discuss, and I can say things that, at this moment, may not make too much sense, but it would be a good idea that you think about them a little bit. We associated states with vectors. States and vectors are the same thing. And it so happens that when you have vectors, you can write them as the sum of other vectors.

So the sum of these two vectors may be this vector. But you can also write it as the sum of these two vectors-- these two vectors add to the state. And you can write any vector as a sum of different vectors, and that's, actually, quite relevant. You will be doing that during the semester-- writing a state a superposition of different things. And in that way you will understand the physics of those states.

So for example, we can think of two states-- A and B. And you see, as I said, states wave functions, vectors-- we're all calling them the same thing. If you have a superposition of the states A and B, what can happen? All right, we'll do it the following way. Let's assume if you measure some property on A, you always get value A. So you measure something-- position, momentum, angular momentum, spin, energy, something-- on A, it states that you always get A. Suppose you measured the same property on B. You always get B as the value.

And now suppose you have a quantum mechanical state, and the state is alpha A plus beta B-- it's a superposition. This is your state. You superimpose A and B. And now you measure that property. That same property you could measure here, you measure it in your state. The question is, what will you get? You've now superimpose those states. On the first state, you always get A; on the second state, you always get B. What do you get on the superimposed states, where alpha and beta are numbers-- complex numbers in general?

Well the most, perhaps, immediate guess is that you would get something in-between maybe alpha A plus beta B or an average or something. But no, that's not what happens in quantum mechanics. In quantum mechanics, you always get A or you always get B. So you can do the experiment many times, and you will get A many times, and you may get B many times. But you never get something intermediate.

So this is very different than in classical physics. If a wave has some amplitudes and you add another wave of different amplitudes, you measure the energy you get something intermediate. Here not! You make the superposition and as you measure you will either get the little a or the little b but with different probabilities.

So roughly speaking, the probability to get little a is proportional to the number in front of here is alpha squared, and the probability to measure little b is proportional to beta squared. So in a quantum superposition, a single measurement doesn't yield an average result or an intermediate result. It leads one or the other. And this should connect with this. Think of the photon we were talking about before.

If you think of the photon that was at an angle alpha in this way, you could say that the polarizer is measuring the polarization of the object. And therefore, what is the possible result it may measure the polarizations say oh, if it's in the x direction you get it right, and what is the probability that you get it to be in the x direction is proportional to cosine squared alpha-- the coefficient here squared. So the probability that you find the photon after measuring in the x direction is closer in squared alpha, and the probability that you'll find that here is sine squared alpha.

And after you measure, you get this state which is to say the following thing. The probability to get the value A is alpha squared, but if you get A, the state becomes A because this whole state of the system becomes that. Because successive measurements will keep giving you the value A. If you get B, the state becomes B. So this is what is called the postulate of measurement and the nature of superposition. This is perhaps the most sophisticated idea we've discussed today, in which in a quantum superposition the results are not intermediate.

So when you want to figure out what state you have, you have to prepare many copies of your state in this quantum system and do the experiment many times. Because sometimes you'll get A, sometimes you'll get B. After you've measured many times, you can assess the probabilities and reconstruct the state.