

MITOCW | L10.2 Transitions with a constant perturbation

Good. So let's do, then, our transitions. So we do the constant perturbation. Constant perturbation. So as we said, ΔH is equal to V , and it's time independent. It just begins at time 0. And we'll examine what's going on by time t_0 .

So what are we going to do? We're going to examine a transition to go from some initial state i , initial state, to a final state f . So we don't have to say much about what the Hamiltonian is or anything. For us, V is going to have a constant. It's going to have some matrix elements that once we do an example you can calculate, but for the time being we need not know too much.

So I'm going to use the key formula that was derived already about perturbation theory and how you get the transition amplitude. So we know that the coefficient c associated to the m state to first order in perturbation theory at that time t_0 can be computed as a sum over all n integral from 0 to t_0 $e^{-i\omega_{mn}t'}$ $\Delta H_{mn}(t')$ over $i\hbar c_n(0)$ dt' .

So let me just say a few words. You may have seen this yesterday in recitation. Here it is. The state has described in this usual language of ψ tilde of t was described as a sum of c_m ψ_m of t ψ_m . And we calculate those in perturbation theory.

So c_n 's, we know them at time equals 0, we imagine we know the initial state and we want to calculate them later. Well, the first order in perturbation theory, the c_n 's are called $c_n^{(1)}$'s. And they're given by this formula. First order in perturbation theory, because we have a single ΔH , this ω_{mn} is the energy of m minus the energy of n over \hbar . And that's it.

Those are all our symbols. So what do we have to do now? We have a transition from some initial state to a final state. So what does that mean? It means that $c_n(0)$, it only exists when n represents the initial state.

So I'll just write Δn_i . At time equals zero, system is in the state i . And at time t_0 , we're asking for the probability to go into final state. So instead of using n and m , we're just using f and i . And therefore, we'll have m equal f .

With these two facts, the formula becomes c_f of the amplitude to first order in perturbation theory to be in the state f at time t_0 is the sum is gone, sum over n just applies for n equals i . So i will go here. We'll have one over $i\hbar$ $\int_0^{t_0} e^{-i\omega_{fi}t'} V_{fi}$, because the ΔH is V , and we're going from initial to final.

So V_{fi} here. And then the t' . It's all gone. It's all become very simple, though. You would say too simple. We have this is time independent, so it goes out of the interval. So we just have the interval of an exponential here.

That's very, very easy. What is V_{fi} case v_i ? So

This goes out, and we just have to integrate this function. I'll write it in a way that is simpler. Maybe you skip a line. I don't want to just count the factors of i and doing the integral. When you integrate this, you get another exponential. You're going to get the exponential at t_0 minus the exponential at zero.

All right, so we don't want to do our integral. So I'll just write the answer. I was saying we get an exponential here at t_0 minus the value at zero, then you take half of the exponential out the form a sign.

This are simple matters, so I will not do the integral here. You get V_{fi} over E_f minus E_i e to the i ω_{fi} t_0 two minus two i sine of ω_{fi} t_0 over two. You can believe that. I think you can believe the sine, and I have everything here. The \hbar helped turn the ω_{fi} into E_f minus E_i . So we can now compute the transition probability to go from the initial to the final stage.

So we'll write it like this. I to f -- it's a little funny. I don't know. You can write it whichever way you want. Some people like it like that. I'm going to do it in the sense that the initial state always appears as a cat, the final state as a bra. So you draw the arrow like that, more or less to keep the sense of order in your brain. But if it doesn't help you, write it whichever way you want. P_{fi} at t_0 one is the norm squared of this coefficient, c_f one.

The probability to be found in the final state has to do with the norm squared of this thing. So it's this that's part of what was reviewed yesterday squared. So what do we get? That simplifies quite a bit. We get V_{fi} squared times four sine squared ω_{fi} t_0 over two over e_f minus e_i squared.

And this is unit free. This has units of energy. V is a variation of the Hamiltonian. It has units of energy. When you put states, states are normalized so it doesn't change the units. And this has units of energy squared, this has no units, and this is the answer. A little strange.

There's a periodic variation on the transition probability, and what does it mean to have a weak perturbation we can ask already? And the answer in general is quite simple. It's a pragmatic answer. A perturbation is weak if this answer is very little, very small. Suppose this probability comes out to be three, you know it's already too big.

But if this is 10 to the minus sixth times this function, that's reasonable. You're shining atoms and one in a million goes and gets ionized. That's a reasonable thing. So the perturbation theory is valid for whatever time you use this formula as long as this number is small, and this could be arranged by having V_{fi} sufficiently small.

I want to understand this function better, because this is a transition from initial state to final state that looks like that. So let's understand it better. Suppose one, E_f is different from E_i , then how does it look? Well, it looks like this. I brought some other chalk not that it helps too much. But it looks like this as a function of time.

The height here is height four V_{fi} squared over E_f minus E_i squared, and it's oscillatory. It goes to zero again at time 2π over ω_{fi} . OK, so this is the oscillation. Actually for a small time, this grows quadratically, and then it starts blowing up. So here while the initial behavior would be quadratic for small time, this actually is quadratic as we will see in a second, but you can more or less see by the expansion of the sine, then the initial quadratic growth gets tamed and becomes an oscillation here.

This is valid for all times if this number is relatively small so that we believe perturbation theory, and that's that for that case. It's also interesting that this gets suppressed as the energy of the final state is different, more and more different, from the energy of the initial state. So it always oscillates, but if the state your transition is very far away, it is going to be extremely suppressed by the quadratic factors.

So this is an important suppression. This is saying that transitions that change the energy are not that favored. A constant perturbation doesn't supply really energy to produce transitions that change the energy much, and they are suppressed.

So they produce them, but they are suppressed. The other case that is of interest is the case when E_f is equal to E_i . I'm not saying that the state f is the same as the state i . Not at all. It's a different state but happens to have the same energy, and in that case, we must take the limit as E_f goes to E_i , remember ω_{fi} is E_f minus E_i so over \hbar .

So the limit as E_f goes to E_i of this P_{if} is how much. I'll kind of do it in my head here. We have an \hbar here that is going to be left over. One over \hbar , so \hbar squared. This is going to cancel.

The four is going to cancel with this, and we're going to get V_{fi} squared over \hbar squared t_0 squared. OK, so here it is, the quadratic behavior when the energy of the final state is the same as the energy of your original state. Now, the transition probability starts to grow quadratically.

That cannot be valid for too long time, because eventually that number grows without bound, and that number could become as big as 1, and that transition is not reasonable. So this is valid up to some max t_0 . And it's up to you, depends on what V_{fi} is, how long you can trust this. So this is a growth.

This is the same growth we observe here. The limit as E_f goes to E_i go to zero is actually the same as the limit as t goes to zero, so it's the same quadratic behavior. So finally, what's going to happen? What are we aiming here? Well, we're aiming to the case where we have in the energy line, we have the initial energy E_i and then we're going to have a continuum of final states that overlap with E_i .

They're all over there. That's E_f all over there. Of course with our box, if you come with your microscope, you see

lines here. But they're all there. There is a continuum overlapping with this, and now we're going to attempt to sum over the continuum.

And what should we observe? We should observe that when we add the continuum physically, what do we need? We need to find what is called a transition rate, in which you have the probability of transition per unit time is a constant. You see, you have a phenom-- you're shining light on an atom. OK, you shine light and you wait. Eventually the atom ionizes-- photoelectric effect.

But if you have a billion atoms, then you can shine light and you're going to have a transition rate, basically how many atoms are going to happen to ionize. So in order to have a transition rate, the probability that you transition has to be proportional to the time that the perturbation has been acting. So the probability of transition must grow linear in t .

Therefore, you have a transition rate which is the probability of transition per unit time, so you can grow linear in time, so per unit time you have a transition rate. So somehow look what's happening here. When E_f is different than E_i , the transition probability is not linear in time. It does this.

When E approaches-- E_f approaches E_i , the probability of transition goes quadratic in time, and what we want is a probability of transition that grows linear in time. That would define a transition rate. So how is that going to happen?

Well, we'll see it happen in front of our eyes. The magic of integration is going to do it. And moreover, we're going to see that consistent with this intuition, most of the transitions that are relevant are happening within Heisenberg's uncertainty principle of a little energy interval here around E_i .

So this will be considered to be at the end of the day energy conserving transitions. The Hamiltonian, the ΔV helps the transition happen but doesn't supply energy at the end of the day. So this is what we're getting to.