PROFESSOR: What are we going to do? We're going to explore only the first Born approximation. And the first Born approximation corresponds to just taking this part.

So this would be the first Born approximation.

It corresponds to what we were doing here. What did we do here? Well, we're simplifying the second term, the integral term, by using what the Green's function looked like. We simplified this term.

So all what we did here was valuable, except that there's one little difference. We-- in that Born approximation, we replaced the psi that appears inside the integral by the incident psi, which is the psi in here. So we will do that now to simplify this quantity in the so-called first Born approximation.

When can we use it? So, very good. So when is the Born approximation a good approximation? Well, we are throwing away terms, in general. When we're putting an expansion of this form, we're saying, OK, we can set the wave function equal to the free part plus the interacting part. So we have the free part, and it gave us this quantity. And the interacting part gave us the second one.

So the Born approximation is good when sort of the free Hamiltonian dominates over the perturbation. So if a scattering center has a finite-energy bump, and you're sending things with very high energy, the Born approximation should be very good. It's a high-energy approximation, in which you are basically saying that inside the integral, you can replace the plane-incident wave, because that dominates.

That's not the whole solution. The whole solution then becomes the plane integral wave plus the scattered wave. But the plane wave dominates over the scattering process. So it should be valid in high energy. It's better and better in high-energy approximation.

So we have it here. And let's, therefore, clean it up. So if we call this equation "equation A," we say, back to A. The first Born approximation, back to A. The first Born approximation gives us psi of r equals e to the i k i r. Now we put all the-- and this is all the arrows. And we have here minus 1 over 4 pi, integral d cubed r prime, e to the minus i k n dot r prime, u of r prime, and e to the i k i dot r prime. All this multiplied by e to the i k r over r.

So let's put a few vectors here. That's it. OK. So it's basically this same thing here, but now replacing the incident wave here. That's the so-called first Born approximation. But now this is really good. We can compare this with what we usually called f theta of phi. The expression of our brackets is the scattering amplitude f theta phi. So here we have an answer. f at wave number k of theta and phi is equal to this integral. Let's write it out. Minus 1 over 4 pi, integral d cubed r prime.

And now we will combine the exponentials. Happily, the two exponentials depend on r prime. So it's a difference of exponents, and we will call it e to the minus i, capital-K vector dot r prime, u of r prime, where this capital-K vector is equal-- I kept the sign-- k n minus-- this time we'll enter with a minus in that k-- k i or k s, the scattering k, minus the incident i vector.

Remember, we defined there, on that blackboard, the scattered momentum as k times the direction of observation, that unit vector. So in combining these two exponentials into a single one, we have this capital-K vector that is a pretty important vector.

And now, this is a nice formula. It kind of tells you story that there are not many ways to generate things that are interesting. Here it says that f k, the scattering amplitude, as a function of theta and phi, is nothing else than a Fourier transform of the potential evaluated at what we would call the transfer momentum.

So the scattering and bridges are doing Fourier transforms of the potential. Pretty nice. Pretty pictorial way of thinking about it. Fourier transforms are functions of-- I think when people look at this formula, there's a little uneasiness, because the angles don't show up on the right side. You have theta and phi on the left, but I don't see a theta, nor a phi, on the right. So I think that has to be always clarified.

So for that, if you want to use, really, theta and phi, I think most people will assume that k incident is indeed in the z-direction. z-direction. So here is k incident. And it has some length.

k scattered has the same length. It's made by the same wave number k without any index, but multiplied by the unit vector n. As opposed to k incident, that is the same k multiplied by the unit z vector. So the scattered vector is the vector in the direction that you're looking at. So this is k s. It's over here.

And therefore this vector is the one that has the phi and theta directions. That is that vector. And the vector k is k scattered minus k initial. So the vector k is the transfer vector-- is the vector that takes you from k initial to k s, is the vector that must be added to k initial to give you the scattered vector.

So this vector, capital vector K-- it's a little cluttered here. Let me put the z in here. That vector is over there, and that vector is a complicated vector, not so easy to express in terms of k i and k s, because it has a component down. But it has an angle phi as well.

But one thing you can say about this vector is its magnitude is easily calculable, because there is a triangle here that we drew that has k incident and k s. And here is k. So this has length k, this has length k, and this has length capital K. The triangle with angle theta. So if you drop a vertical line, you see that k is twice this little piece, which is little k sine theta over 2.

So that's one way that formula on the right-hand side has the information of theta. It also has the information of phi, because you also need phi to determine the vector k.

So this is an approximation, but look how powerful approximations are, in general, in physics. This approximation is an approximation for the scattering amplitude. It-- first, it's a very nice physical interpretation, in terms of a Fourier transform of potential. Second, it gives you answers even in the case where the potential is not spherically symmetric.

You remember, when potentials were spherically symmetric, the scattering amplitude didn't depend on phi, and we could use partial waves. And that's a nice way of solving things. But here, at the expense of not being exact, we have been able to calculate a scattering amplitude when the potential is not spherically symmetric. So you manage to go very far with approximations. You don't get the exact things, but you can go into results that are a lot more powerful.

So this is an explanation of this formula. And we can use this formula. In fact, we'll do a little example. And these are the things that you still have to do a bit in the homework as well. Many of them are in Griffiths. And indeed, technically speaking, what you need to finish for tomorrow hinges a bit in what I'm saying. But the final formulas are well written in Griffiths explicitly, and in fact, half of the problems are solved there. So it shouldn't be so difficult.