PROFESSOR: OK. So what is our Schrodinger equation? Therefore, our Schrodinger equation is ih bar d psi dt is equal to 1 over 2 m. H bar over i grad-- that's p-- minus q over c a squared plus q phi on psi.

We're going to motivate that next time. But let's look at it for a little while, at least. There's several things we've done here. We've replaced p by p minus q over c a. So this is a replacement. P has been replaced by that. We used to have p squared over 2m. Now we have this quantity. Well, we'll see why that is the right thing to do, but you could ask yourself, is this p still intuitively equal to mv or not?

And the answer is no. That's not really mv. We will see that from Heisenberg's equation of motion, but intuitively, when you have here-- this is the energy of a particle-- kinetic energy and potential energy-- the kinetic energy-- 1 over 2m times m squared v squared. That's p squared. So 1 over 2m m squared v squared is 1/2 mv squared. That's the kinetic energy. And that should come from here.

So what we will see is that, if you want to speak about the velocity operator, this whole thing is the velocity operator. We don't speak, really, of velocity operators in quantum mechanics before we put electromagnetic fields, but here, it will be natural to call this m times the velocity operator in the sense of Heisenberg equations of motion.

This operator is Heisenberg equation of motion-- is going to look like the Lorenz force equation. So it will be reasonable to think of this like that. This operator is nicer than the operator p, as we will see, for many reasons.

So we've emphasized gauging variance so that, perhaps, the most important thing we could say, now, to end up this lecture is, what is the statement of gauge invariance for this Schrodinger equation? So gauge invariance means that the physics that you obtain with one set of potentials should be the same as the physics you obtain with a gauge-equivalent set of potentials.

So I will say this way-- suppose you solve the Schrodinger equation with h bar over i grad minus q over c, with the new potentials-- plus q-- or you solve the Schrodinger equation with the old potentials?

So you have here the two Schrodinger equations-- one with the new potentials, one with the old potentials. They should be the same physical solution. This is not going to be too obvious how to do, however. How do I guarantee they are the same physical solution? I'll have to go on a limb and try something.

Look. Here are the gauge transformation. That's what a prime is. That's what phi prime is. Should the same psi be a solution of both? Should this equation imply this equation, so that the same psi works when you change the gauge potentials? That would not work. That is asking too much. Certainly, the same psi worked would be simple looking, but that's not what you really have to demand. It's not going to be able to occur here.

What you're going to need is to change psi as well. The gauge transformation is going to affect the wave function, too. Not only the electromagnetic fields get gauge transform-- the wave function must be gauge transform. And you would say, OK. That sounds a little dangerous because if you change the wave function, you're going to change the physics. Could happen.

But the change in the wave function is going to be subtle enough-- is going to be just by a phase. That can still change the physics. If you have a complex phase, you can change the physics. But will be simple enough that we will check that the physics is not changed. So the claim of gauge invariance is a statement that this equation implies this, or this implies that, if psi prime also transforms. And the formula is-- at psi prime-- should be equal to e to the i q over hc lambda times psi.

So that's the key to it. When you transform the potentials-- when you change a to a prime and phi to phi prime-- you should change psi to psi prime. What? With what? Using the same lambda that you needed to change the potentials, you do a phase rotation. And it's not a constant phase. This depends on x and t.

So it's a substantial change. So you now have the technical problem of first checking that this is true. This is the statement of gauge invariance of the Schrodinger equation. There is a way to transform the wave function so that the new Schrodinger equation solution is obtained from the old Schrodinger equation solution. And then we will have to check that the physics is the same.

If you wanted to compute the expectation value of x, on this wave function, this phase factor would cancel. So it would give you the same. If you want to compute some other expectation values, it's a little funny. So there will be operators that are nice for wave functions or [INAUDIBLE] gauge invariant operators.

And it will be a nice story that we will develop next time.