MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II — October 2, 2012
Prof. Alan Guth

PROBLEM SET 4 REVISED*

DUE DATE: Friday, October 5, 2012. Either hand it in at the lecture, or by 6:00 pm in the 8.07 homework boxes.

READING ASSIGNMENT: Chapter 3 of Griffiths: *Special Techniques*, Secs. 3.3-3.4.

PROBLEM 1: LAPLACE'S EQUATION IN A BOX *(15 points)*

Griffiths Problem 3.15 (p. 136).

PROBLEM 2: A SPHERICAL CONDUCTOR AND A CONDUCTING PLANE *(25 points)*

Consider a solid spherical conductor of radius R, with center on the positive z-axis at $z = z_0$, with $z_0 > R$. Suppose that the x-y plane is conducting, and is held at potential $V = 0$, while the sphere is held at potential V_0.

To first approximation, we can think of the field as that of a point charge q_0 at the center of the sphere, with q_0 related to V_0 by

$$V_0 = \frac{q_0}{4\pi\epsilon_0 R} \ . \tag{2.1}$$

The field due to this charge gives a potential V_0 on the surface of the sphere, as desired. But now the potential on the x-y plane is not zero.

(a) The potential on the x-y plane can be restored to zero by placing an image charge below the x-y plane (i.e., at negative z). What charge q' should this image have, and where should it be placed?

(b) The potential on the surface of the spherical conductor is now no longer constant, but it can be made constant by adding another image charge q''. The potential on the x-y plane can be restored to zero by adding another image charge q''', and the potential on the sphere can be restored to a constant by adding yet another image charge q''''. The series will continue forever, but it does converge fairly quickly. Calculate the positions and charges of the image charges q'', q''', and q''''.

(c) After all the image charges are added through q'''', what is the potential V of the spherical conductor?

(d) What is the total potential energy of this configuration? Express your answer as the first terms of an infinite series, showing those terms corresponding to the image charges through q''''.

(e) Would the fields outside the conductors be different if the solid spherical conductor were replaced by a spherical conducting shell, with the same outer radius?

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* The wording of Problem 2(d) has been changed since the original version of September 29, 2012.

PROBLEM 3: ELECTROSTATICS INSIDE A SPHERICAL CAVITY *(15 points)*

(Patterned after Jackson Problem 3.5.)

A hollow sphere of inner radius a has the potential specified on its surface to be $V = V_0(\theta, \phi)$. Show that the potential inside the sphere can be written as

$$V(\vec{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm} \left(\frac{r}{a}\right)^l Y_{lm}(\theta, \phi) \ , \tag{3.1}$$

where you are asked to find the expressions for A_{lm} in terms of $V_0(\theta, \phi)$.

PROBLEM 4: A CHARGED METAL SPHERE IN A UNIFORM FIELD *(15 points)*

Griffiths Problem 3.20 (p. 145).

PROBLEM 5: A SPHERE WITH OPPOSITELY CHARGED HEMISPHERES *(15 points)*

Griffiths Problem 3.22 (p. 145).

PROBLEM 6: AVERAGE FIELD INSIDE A SPHERE *(20 points)*

Griffiths Problem 3.41 (p. 156).

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