

[SQUEAKING]

[RUSTLING]

[CLICKING]

**PROFESSOR:** Welcome back to 8.20. In this section, you want to look at light, what is it, and how does it propagate.

In this video, specifically, I give you a little bit of a preview of 8.02. And I don't do this in a very topological way. I just give you some information.

So if we study 8.02, we'll see Maxwell equations are being developed in there. We look at Maxwell equations for electric and magnetic field  $E$  and  $B$  in vacuum.

We can rewrite the Maxwell equations and define wave equations. The solutions of the wave equation, as the name tells you, are waves.

So what we are looking at here is you want to describe the propagation of electric and magnetic fields in vacuum. In this situation, this is maybe at some time,  $t$  equal to 0, we have an electric field in this point here, and a magnetic field-- electric field points into the  $y$  direction, the magnetic field into the  $z$  direction.

And what the equations now describe is how the wave propagates in space and in time. And you can already tell from the name, wave equation, the solutions of this equation-- these differential sines and cosines.

So one solution here is  $E_y$  equal to  $E_0$ , times cosine,  $kx$  minus  $\omega t$ . We find then that the speed in which the wave propagates-- you pick one peak of a wave, and you see how it propagates-- one point of variance here propagates. The speed in which it propagates is the speed of light,  $c$ .

And you can find  $c$  here through those constants in the Maxwell equations and wave equations. Find  $c$  is  $1$  over square root  $\epsilon_0$  and  $\mu_0$ . The permeativity and the permeability, the product of the two gives you the speed of light.

So if you look at this some more, and connect the Maxwell equation to the Lorentz force, again, as a reminder, for those who had had 8.02 already, the force on the charged particle in an electromagnetic field is given by  $qE$ , plus  $q\mathbf{v} \times \mathbf{B}$ .

If you have two charges, the force between those two charges is the product of the  $q$  divided by  $r$  squared, times  $1$  over  $4\pi \times \mu_0$ . Again, [INAUDIBLE]. And the force between two wires-- this current-- current  $i_1$  and current  $i_2$ -- is equal to the product of the two currents, divided by  $r$ , times  $l$ -- the length of the wires-- times  $\mu_0$  over  $2\pi$ .

So this is fantastic, because now you can calculate the speed of light by just measuring the forces between charges and current in wire's centers.

The value of  $c$  is also very interesting. It's large-- very large.  $3 \times 10^8$  meters per second. So just let that sink in. We, as humans, move with a few meters per second. Light travels-- a few nanoseconds is needed for light to travel about 1 meter. It takes just nanosecond.

Let's stop the video here. The next thing I want to do is an exercise. I want to have you play with this differential equation, and there's a solution of the differential equation.

But the challenge or the exercise is to show that if you have a function which you can write as  $f_0$ , which is an arbitrary function, which is a function of  $x - ct$ , those functions, regardless in how they look like, are solutions of this differential equation.

Note that I replaced our constant  $\epsilon_0$  and  $\mu_0$  now with  $1/c^2$ . So  $f_0$  can really be an arbitrary function. You need to be able to build the derivative, though.

So I do the function here as a function of  $x$  for some time equal  $t_0$ . And then I drew the same function 4 times equal to 1. And so you can, from this picture, see that the  $\Delta x / \Delta t$  is minus  $c$  in this case. So my function-- my wave is moving with the speed of light in minus direction.

So I want you to show that this kind of equation [INAUDIBLE] wave equation. So I would like you to do this, and stop the video, and show you the solution next.

So the way to approach this is simply applying the chain rule. And that might be something you want to remind yourself of. So after I do this, I'll define this little helper function here  $u$  is equal to  $x - ct$ .

And this makes our function a function of  $u$ , which is itself a function of  $x$  and  $t$ . So if I built a derivative with  $x$ , I have this  $df/du \cdot du/dx$ .

If I build a second derivative, there is a product you have to take care of. So I find that  $d^2f/dx^2$  is the second derivative of  $f$  of  $u$  here, times  $(du/dx)^2$ . And then I have to add  $df/du$  times second derivative of  $u$ .

This follows very similar for the derivative of  $t$ . And then I can investigate what we find. So my  $du/dx$  is equal to  $1 - c \cdot dt/dx$ .

If I build the derivative of  $x - ct$ , this  $x$ , I find 1. I do the same with  $t$ -- I find minus  $c$ . I will use this second derivatives of  $u$ .  $x$  and  $t$  are all 0.

If I put this now in my equation, I find second derivative of  $f$  with  $u$  is of  $1 - c^2$ -- sorry,  $1 - 1/c^2$  times  $c^2$  is equal to 0. And since this is always 0, we have just proven that any sort of function which I can build the derivative of which is of the form  $x - ct$  solves that equation.