MARKUS KLUTE: Welcome back to 8.20 Special Relativity. In our quest to understand how we get to general relativity, there is two things to consider. The first one, this lecture is not meant to give you a full description of general relativity, but just a view into where this might lead, where this discussion might lead.

So in this quest, we can understand the theory of general relativity as a theory on how to patch together the different reference frames which each can be described in special relativity, in the framework we discussed up to now, and it's valid in short intervals in spacetime. Consequences of general relativity are that spacetime is curved.

So we have modified geometries. We learned that, because of gravitational effects, matter curves spacetime. As a consequence of that, there must be modification of gravity based on matter distributions, and so there must also be gravitational waves, gravitational lenses which bend light, black holes, and there's cosmological predictions coming out of general relativity.

So let's have a discussion first. What does it mean to have a changed or modified geometry? What could that mean? So you are all used to Euclidean geometry, where, when you draw a triangle, you add up all the angles to 180 degrees. If you try to parallel lines that never cross, they also don't diverge.

But if you have a modified geometry-- for example, the geometry on a sphere, like on our globe-- the angles do not add up to 180 degrees. Actually, the sum is larger than 180 degrees, and parallel lines will cross. We will call this kind of space positively curved, but you can have the opposite example, like on a saddle.

So you can have other spaces and other curved spaces, and they can be negatively curved. In this example, if you add up all angles, you find they add up to less than 180 degrees. Parallel lines do not cross, but they will diverge. Mass changes the geometry of spacetime. We just talked about light bending, and because of the change in geometry, light will not go on a straight line anymore, but will bend around massive objects.

Spacetime is curved. Geometry of spacetime tells us how the mass is moved. You can think about a trampoline. When you put a heavy object on a trampoline, all the other objects on the trampoline will gravitate towards the heavier object, and that's kind of a picture on how spacetime actually looks like.

Einstein used those findings in order to redefine Newton's first law and found the so-called Einstein field equation. So on one side of the equation, there's a description of spacetime and its curvature, and on the other side of the equation is the energy momentum tensor, the description of how energy and momentum of object is distributed.

And those two things, spacetime and energy and momentum, they're kind of interlinked in this equation. So if you read this description, you can read it from one side to the next. Spacetime tells matter how to move. Or you read from the other direction, say matter tells spacetime how to curve. That is an equation, and you can just read it from the left to the right or from the right to the left.

Our understanding here. It says space and time are not fixed things through which matter and energy moves through. The matter and energy themselves define spacetime. And matter, because of spacetime, is dynamical. It's changing. It's interacting with the matter and with the energy.

This is a super exciting picture from Hubble, the Hubble Space Telescope. And you see galaxies, but what you also see is those structures which looks like the light has come through lenses. Those lenses are actually matter distributions, galaxies, which actually lead to the bending of the light and those lensing effects.

OK. If you want to summarize general relativity, you can first say that spacetime is curved and it follows the pseudo-Riemannian manifold with a specific metric. We have seen the metric before. It's minus, plus, plus, plus. And the relationship between matter and curvature is given by the Einstein equation, and here I give you a slightly different form where there is the dynamics, again, on one side and the energy momentum on the other side.

Let's just look at one example here. So we discussed, in special relativity, invariant intervals, and we have this delta squared, or we have a different name for it. I given by minus dt squared plus dx squared plus dy squared plus dz squared, and we could have just written this in polar coordinates as well, where you find that dr squared and r squared d theta squared, and then r squared sine squared theta d phi squared. OK. Same thing. It's just a different coordinate system.

So as a solution to Einstein equation, we find something which looks very, very similar. That's not a surprise, as we find general relativity as a patchwork of small spaces of special relativity. So the solutions might be very similar. And the solution found here, the so-called Schwarzschild solution, which is a unique solution in vacuum with spherical symmetry of a matter distribution.

So you have a spherical matter distribution like our sun, and this is a solution which describes spacetime around this. You find this invariant interval here has two interesting features. There's two singularities in here. This should be a minus 1. You find those two singularities. One is at r equals 0. That's kind of expected. In the middle of the mass distribution, this thing is not defined anymore. There's no mass left.

But there's also a second singularity at 2GM. This is called the so-called Schwarzschild radius, and if you get to the singularity, you basically don't define anymore this invariant interval. You can think about the surface of a black hole as this singularity. At this r value, at the singularities, everything becomes timelike, or everything within the radius becomes timelike.