MARKUS KLUTE: Welcome back to 8.20. In the previous section, we have seen the relativistic Doppler effect, and now we want to study how light-- in this case, a monochromatic plane wave-- transforms in the Lorentz transformation. In other words, we have, for example, a distant star emitting light at a specific frequency. The question now is, how do we observe this light when the star is traveling away from us or towards us?

So here you see our monochromatic plane wave. We have an amplitude A and then just a simple cosine, which is a function of x, y, and t, time. This is the solution of the wave equation, and we have already seen this as part of the p sets, but also discussed in class.

So the wave is characterized by so-called wave numbers in x direction and y direction. The squared sum-- the square root of the squared sum is the so-called wave number. The frequency omega is equal to 2 pi f, where f is the frequency and omega is the angular frequency. And if you divide the angular frequency by the wave number, you get the speed of light. Similarly, you can multiply the frequency and the wavelength.

OK, so as a first activity, I will ask you to see that how does this solution, how does this specific wave, transform on the Lorentz transformation? As a reminder, we have seen that the equation which governs how this light propagates is [INAUDIBLE] the Lorentz transformation. But now we want to investigate what happens to the wave itself. OK?

So we have to investigate this specific solution and Lorentz transform x and t. And I just do this here in this equation. So you see that now we have, as part of the cosine, Kx gamma x prime plus beta ct prime plus no change in y direction as we look at the Lorentz transformation in x direction, and then we have the transformation of the time axis. OK?

So now this looks very cumbersome or complicated, but we can try to refind the very same characterization of the wave as we had before. How does now the transformed wave number look like or does the frequency look like after Lorentz transformation? And so we want to identify the individual terms Kx prime, where we label Kx prime as the parameter you find here in the solution, in this Lorentz transform solution, and we do the same for omega prime, and you find a solution here.

So now there's this angle cosine I defined as the angle with respect to the line of motion. Omega prime is now the baseline frequency and omega the one which is detected. That's just a matter of changing the direction of beta with the plus and minus sign. But if we use that definition, we can now discuss the result.

So as part of the discussion, we can look at the specific case where the wave is moving towards us. OK? So zeta is equal to 0 and beta is positive. In this case, omega is larger than omega prime. And so the frequency is going to be higher. So the detected frequency is going to be higher blueshifted.

So if you have a situation that a star is moving up towards us and emitting light, the light is detected by us, maybe by our eyes or by a telescope, that light is going to be blueshifted. It's going to go to higher frequencies. The opposite scenario is where zeta is equal to 180 degrees and beta equal to-- greater than 0, or the other way around. We could have defined this also as zeta equals 0 and beta negative.

In this case, omega is smaller than omega prime. So the frequency is lower, meaning that the light we observe is redshifted. And therefore, this term redshift is a measure of whether or not the source of light is moving towards us or away from us. And the larger the redshift, the higher the velocity is of this object moving away from us.

So we can define this redshift as the relative change in frequency omega prime minus omega over omega, or we can define 1 plus Z, 1 plus the redshift is omega prime over omega, which is square root of 1 plus beta over 1 minus beta. All right?

So if you now observe the stars in our galaxy, and you can do this, for example, by its specific spectral form. There are the specific spectral lines, lines of specific frequency, which we can observe from stars as they are in certain distance from our solar system. And if we do this, we basically see all stars being redshifted, meaning all stars are actually moving away from us, which is a measure of the fact that the universe is expanding.