Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
Term:	IAP 2021
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Problem Set 3

handed out January 19th, 2021

Problem 1: "This Ion Cannon goes to 11..." [20 pts]

A Separatist vessel is attempting to escape from a Republic cruiser (sorry, this is somewhat regular dinner conversation ...) in hot pursuit. The separatist ship is traveling at a velocity $\frac{2c}{3}$ while the cruiser is traveling at a velocity of $\frac{c}{5}$ (both with respect to some "stationary" observer located on a nearby ship).

(a) The cruiser launches a blasting rocket which travels at $\frac{c}{5}$ with respect to the cruiser ship. What speed does the rocket achieve and can it catch up to the vessel?

(b) But wait! The blaster rocket itself fires another ballistic missile, also at a speed of $\frac{c}{5}$ with respect to the rocket. Does *that* missile hit its target?

(c) Analyze the above situation, but this time from the point of view of the cruiser. Does the rebel ship still manage to escape?

Problem 2: The Ladder Paradox [20 pts]

Suppose a farmer living in Mr. Thompson's Wonderland[1] using his unicycle (see figure) wishes to store his ladder in his barn (which is much shorter than the ladder).

[1] *Mr. Thompson's Wonderland* refers to a 1959 paper by Terrell whereby the speed of light is much smaller and hence the effects of special relativity are readily apparent.

Having studied relativity (but only for the 1st week) he reasons once the ladder is at high enough speed, it should Lorentz contract and easily fit within the door. Once the back of the ladder has cleared the door, he will shut the door, stop the ladder and lock it inside. A neighbor politely points out that from the ladder's vantage point, it is the barn that will shrink. Who is right?

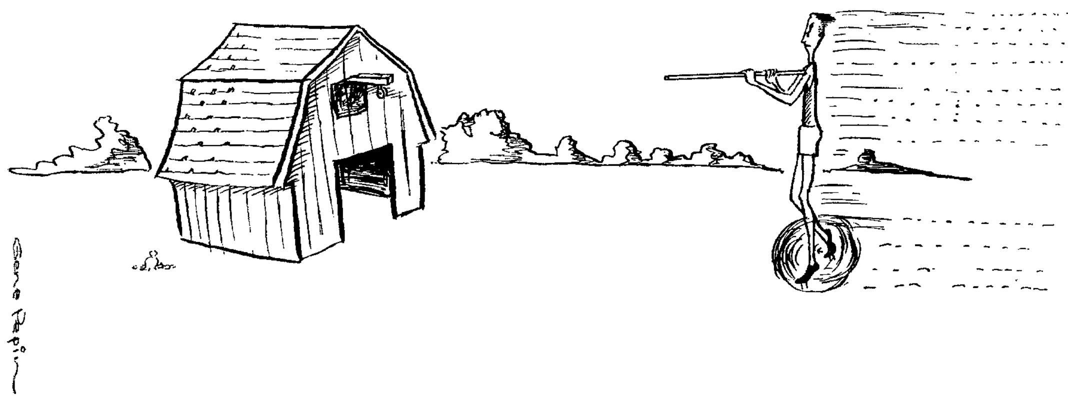


Fig. 1. Pole vaulter running toward open front door of barn with contracted pole.[6]

Problem 3: The Paradox of the Fast Walker [20 pts]

This is a slight twist from the ladder paradox problem, offered by Wolfgang Rindler (Am. J. Phys. 29, 365 (1961)). A fast walker is walking over a metal grid whose's spacing is exact the length of his foot. From his point of view, the crate spacing is Lorentz contracted, so he should easily be able to walk over the grid. But from the grid's frame of reference, it is the foot which is Lorentz contracted, and he will surely fall in. Explain what happens.

Problem 4: Velocity Addition in Matrix Form [20 pts]

The Lorentz transformations allow one to correctly transform from one inertial frame to the next. In Lorentz transformations, spatial and time coordinates are intertwined, where one is no longer independent of another. As a reminder, the Lorentz transformation to a frame moving with velocity v along the x-axis is given as the following:

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \end{aligned}$$

where $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$. These transformations can be represented more compactly in matrix notation.

(a) Begin by writing down the *Galilean* transformation from a frame S to a frame S', where S' is moving with a constant velocity v along the x-axis, in matrix notation. Use ct for the time component.

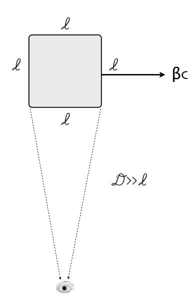
(b) Now write down the Lorentz transformation for the same situation as above.

(c) Use the property $\gamma^2 - \gamma^2\beta^2 = 1$ to re-write the above matrix in terms of $\cosh\phi$ and $\sinh\phi$, where $\tanh\phi = \beta$.

(d) Allow $\Lambda(\phi(\beta))$ to represent the matrix that you have written above. Show that $\Lambda(\phi(\beta_1))\Lambda(\phi(\beta_2)) = \Lambda(\phi(\beta_1)+\phi(\beta_2))$. Is this consistent with the velocity addition rule?

(e) From the above, show that the inverse of $\Lambda(\phi(\beta))$ is given by $\Lambda(\phi(-\beta))$.

Problem 5: Rotating Cube [20 pts]



Suppose a cube whose sides are length l (in the cube's proper frame) travels to the right at velocity βc with respect to the lab frame. An observer is located in the lab frame and watches the cube pass by. We say the "left" and "right" faces of the cube are perpendicular to the direction of motion, while the "front" and "back" faces are parallel. The cube's distance of closest approach is much, much greater than the length of the cube.

Because the light-path from corners of the "back" and "front" faces of the cube to the observer differ, the cube will appear to be rotating. What is the angle by which the cube appears to be rotated?

[*Hint*: Everything we perceive are merely projections of three-dimensional objects on the two-dimensional plane of our retina. With this in mind, how would we perceive a rotated cube to be different from one that is not? How would the finite speed of light and other relativistic effects contribute to trick us to perceive the cube as rotated?]