Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
Term:	IAP 2021
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Final Examination

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Task 1: Short Questions [10 points]

Determine if these quantities satisfy one or more of the following conditions:
(A) invariant under Lorentz transformations,
(B) invariant under Galilean transformations,
(C) conserved (within the context of Special Relativity), or
(D) none of the above.
Give all correct answers.

(1) (Coordinate or Ordinary) Time.

(2) Proper Time.

(3) Rest Mass.

(4) Force (defined as mass times acceleration).

(5) Speed of a massless particle.

(6) Electric Charge.

(7) Kinetic Energy.

(8) Speed of light in vacuum.

(9) The wavelength of light emitted by a cesium clock.

(10) The angle at which a distant object is observed.

Task 2: Short Questions [10 points]

Answer the following questions *briefly*.

(a) In what limit can a Lorentz transformation be approximated as a Gallilean transformation?

(b) Are moving rulers shorter or longer for an observer at rest?

(c) An object is moving at $v = 9/10c$. As it moves past, you measure that in your frame of reference it is perfectly spherical. In its own reference frame is it (i) a sphere, (ii) a prolate ellipsoid (with one axis longer than the other two – like an American football), or (iii) an oblate ellipsoid (with one axis shorter than the other two – like an M&M candy)? Justify your choice.

(d) When changing reference frames moving with a relative velocity in x-direction, does the velocity of an object in direction perpendicular to the x-direction change?

(e) In the scattering of a photon with an electron (Compton scattering), there must be a change in the photon energy if the scattering angle in non-zero. Explain.

Task 3: SN1987A Neutrinos [20 pts]

Figure 1: Photographs of supernova 1987A before (left) and after (right) the explosion. The object was 52 kpc away and emitted copious amounts of neutrinos during the explosion.

On February 23rd, 1987, a supernova was detected to have exploded near the Large Magellanic Cloud. A giant burst of light was seen in the southern sky but, more remarkably, 3 hours before a sudden burst of neutrinos was detected at several neutrino detectors around the world. The supernova was located about 50 kiloparsecs (1.5×10^{21} meters) from Earth. The time spread of the neutrinos detected was less than 15 seconds and their energy was on average 10 MeV (where 1 MeV = 10^6 eV). From this information you will provide an upper limit on the mass of the neutrino.

(a) Begin by solving for the amount of time it takes for a particle of mass m and energy E to traverse a distance d. Recall that the ratio of momentum and energy yields the speed of the particle.

(b) With the above expression, expand to second order of mc^2/E to obtain a function that relates the time of propagation to the neutrino mass.

(c) Assuming a massless particle would travel in time $t_0 = d/c$, use the spread in arrival times to provide an upper limit on the neutrino mass. Express your answer in terms of eV/c^2.

(d) The OPERA neutrino experiment measured (by mistake) the neutrino velocity slightly higher than the speed of light, by about $\delta c/c \sim 2 \times 10^{-5}$. If this difference in speeds persists for neutrinos created in the supernova, what would have been the time difference measured between the light signal and the neutrino signal.

Task 4: Creating Top Quarks [20 points]

The top quark is an elementary particle with the rest mass m_t which is about 175 times the mass of the proton, m_p. Particle physicists produce and studied top quarks at the Fermi National Accelerator Laboratory (Fermilab) via the reaction:

$$p + \bar{p} \rightarrow t + \bar{t}$$

Here, p is a proton, $\bar{p}$ an antiproton with the same mass as the proton; t is a top quark and $\bar{t}$ is an anti-top quark with the same mass as the top quark.
The mass of the proton is 0.938 GeV/c^2, but for this problem let us simplify the numbers by taking $m_p = 1$ GeV/c^2.

(a) Physicists at Fermilab have built two accelerators. One accelerates protons to an energy E; the second accelerates anti-protons to the same energy E. What is the minimal value of E to produce a t and $\bar{t}$ in the collision? What is the value of γ of the incident protons and anti-protons?

(b) Now, consider making top quarks by accelerating anti-protons and colliding them with protons which are at rest in the lab frame. To what γ, and hence to what minimal energy, must the anti-proton be accelerated in order to make a t and $\bar{t}$ in the collision?

Task 5: Measuring the Higgs boson [20 points]

Imagine that, as a result of all your lobbying of Congress and the President, you are put in charge of designing a new collider dedicated for precision measurements of the Higgs particle. You have a choice of building an electron-positron collider ($e^+\ e^-$), a muon-anti-muon collider ($\mu^+\ \mu^-$), or a photon-photon collider ($\gamma\ \gamma$). The relevant information about these particles are given in the table below.

Table 1: Particle properties

Particle	Mass	Charge	Lifetime
Electron/Positron (e^+/e^-)	0.5 MeV/c^2	±1	Stable
Muon (μ^+/μ^-)	105 MeV/c^2	±1	$2.2\ \times10^{-6}$ s
Photon (γ)	0	0	Stable
Higgs (H^0)	125 GeV/c^2	0	10^{-22} s

(a) Suppose you wish to create a single Higgs from your collider at the estimated mass of 125 GeV/c^2. What kinetic energy must be given to each particle under the different scenarios to reach the said goal (you may leave as a variable or estimate numerically).

(b) Suppose you choose (for political reasons) to try the $\mu^+\mu^-$ collider. How much time do you have to transport the muon from where it was created to its collision point before it decays away? Give your time answer as viewed in the lab frame.

(c) Your arch-nemesis, Dr. Zoidberg, lobbies against you and claims that it would be far better to build these machines as fixed target (i.e. lab-frame collision) machines. Give arguments against the case (I can think of at least one for each particle type).

Task 6: How long before the Sun runs out of fuel [20 points]

The sun's power output is due to nuclear reactions occuring deep within it with the net effect of turning four protons into an alpha particle, two anti-electrons, two neutrinos and some photons. The anti-electrons then meet electrons and annihilate, yielding more photons. All of the photons (those produced in the intitial reactions and those produced when anti-electrons annihilate) serve to heat the core of the sun. The kinetic energy of the alpha particles also contributes to heating the core of the sun. The neutrinos escape from the core of the sun without heating it. The two neutrinos produced with each alpha particle carry off an energy of about 0.8 MeV. With the exception of this 0.8 MeV carried away by the neutrinos, all the energy released by turning four protons into one alpha particle ends up as heat. The rest mass of a proton is 938.3 MeV/c^2. An alpha particle, also known as the nucleus of a ^{4}He atom, has a rest mass of 3727.5 MeV/c^2.

(a) For each alpha particle which is produced, calculate the amount of heat energy which is produced. What fraction of the rest mass of the four protons is converted into heat?

(b) The core of the sun is not becoming hotter. Instead, the heat produced at the core diffuses outward and is radiated as light from the surface of the sun. The total energy of the light emitted by the sun in one second is 3.8×10^{26} Joules. By how much does this emission of light decrease the rest mass of the sun each second?

(c) The mass of the sun is 1.99×10^{30}kg. At present, approximately 3/4 of its mass is protons, and approximately 1/4 of its mass is alpha particles. Assuming that the luminosity of the sun does not change over time, how long will it take for the sun to convert all its protons into alpha particles?

[**Aside:** The sun will actually not shine by converting protons into alpha particles for as long as you have calculated. There are two reasons your calculation yields an overestimate. First, the luminosity of the sun is not constant; it increases somewhat over billions of years. Second, and more important, the properties of the sun change drastically when all the protons in the core are gone. At that time, the sun begins generating heat by turning alpha particles into carbon nuclei, and becomes what is known as a red giant star. At that time, there are still a lot of protons present in the outer parts of the sun. Therefore, the sun will turn into a red giant at a much younger age than the estimate you have made by assuming that it continues to shine by turning protons into alpha particles until it has converted all its protons into alpha particles.]

8.20 Introduction to Special Relativity
IAP 2021