MARKUS KLUTE: Welcome back to 8.20, special relativity. We're going to continue our discussion of galactic space travel. Here the situation is slightly modified from the previous one. We still have Alice being our ground control and Bob riding on a spacecraft in order to explore planetary systems and solar systems.

This situation is different in the sense that the spacecraft has an escape rod. So it's able to send probes to planets in order to study them. And so, in this specific case, the velocity of this escape rod is uB delta xB over delta tB, as measured in Bob's reference frame. The direction of this velocity is the same as the direction of Bob's spacecraft when looking in longitudinal direction.

Again, as a reminder, velocity is distance over time or delta x over delta t. So the question now is, what does Alice observe? Because this is an activity you should try to work out yourself, stop the video. I'll just continue here.

So, if you calculate now the velocity as seen by Alice, delta xA over delta tA, that's given by gamma times delta. We just use Lorentz transformation-- gamma times delta xB plus v times delta tB over gamma times delta tB plus v over c squared delta xB.

Now, we can cancel the gammas and take out delta tB out of the brackets. And then we find uB plus v over 1 plus v over c squared uB. So this looks like an addition of velocities with a correction factor of 1 plus v over c squared uB. Good.

So does this make sense? So, whenever we have a calculation like this, we should check that it actually works out, that extreme cases are preserved, and that units work out. So, again, the units work out here. On both sides, you find meters per second, the unit of velocity.

If you check now what happens if we set uB equal to 0, we know that the escape rod is at rest. There's no velocity with respect to Bob's reference frame. In that case, we find that uA is equal to v, exactly the velocity difference, the relative velocity between those two reference frames.

If that velocity is 0, we find uA equal to uB. Again, that's expected. If Bob and Alice are in the same reference frame and they observe the same escape rod, they better measure the same velocity.

And, lastly, if we now, instead of having an escape rod, we send a beam of light out, which has a speed-- a beam of light has the speed of light, uB equal to c, we find that the velocity observed by Alice is also c, which brings us to an interesting point here. Yes, we still add velocities with a little bit of a relativistic correction, but we will never get larger velocities of the speed of light. So the speed of light is an absolute speed limit.

Let's analyze this a little bit more in the context of our light clocks. So what now happens if the velocity is equal to c is that gamma goes to infinite?

And, in the context of the light clock, you can notice that the upper mirror can never be reached. It's moving with the speed of light, the same velocity as the light itself. So light is never able to reach this. The clock will stop. All right, so there's an absolute limit of velocity at the speed of light.

OK, so now, so far, we discussed only velocities in the direction in which the two reference frames move or the second reference frame moves with respect to the first one. What now happens if we consider perpendicular velocities? So, in this case, Bob's spacecraft, this escape rod goes up. Maybe he's circling the planet, or he's just approaching the planet. And [INAUDIBLE] when it [? pops at ?] that planet specifically.

So here we want to work out the example in which the perpendicular velocity is not 0, but the longitudinal velocity is 0. So what does Alice observe? So we do this as a concept question. Which of the four answers is correct?

Is the velocity unchanged because we are studying perpendicular velocity? Is the velocity smaller, larger, or you don't know because you actually have to figure it out, work it out?

OK, so the velocity, as observed by Alice, is actually the absolute value is smaller than the one observed by Bob. We can do the very same calculation. So we have uB y is delta yB over delta tB. And then, for Alice, this is uA y delta yA over delta tB.

So the y-component, the length measured in y-direction between Bob and Alice, is invariant, as we saw in the previous section, but the time is not. So we do have to do the Lorentz transformation of delta tA and find that, in the case where uB x is equal to 0, that we just have to divide uB y over gamma.

Situation is a little bit more involved when there's also a longitudinal velocity, but you see here how this would unfold. So 2 was the correct answer here. So, while the length in longitudinal directions are invariant, the velocities are not. And that's because time is suspect. Time needs to be corrected in the two reference frames.