MARKUS KLUTE: Welcome back to 8.20, special relativity. In the previous section, we have seen how we can look at energy and momentum of particles in a decay. Here we now want to, in collisions of particles, create new particles.

The example, the first example here, is the collision of two protons to create a proton, a neutron, and a charged pion. The masses are given there. So the question now is, what is the minimal energy needed in order for this process to occur in a fixed-target experiment?

Fixed-target experiment is we have an accelerated proton and another proton at rest. This might just be a hydrogen target just sitting there. So the question is, how much energy-- how much do we have to accelerate the proton for this process to be possible? Now, again, stop the video, and try to work this out.

The important part here is to realize that minimal energy here means that, after the decay or the decay after the process occurred, all the new particles need to be addressed. That is when the process requires minimal energy. So, instead of analyzing this in the laboratory frame, we want to analyze this in the center-of-mass frame.

All right, the momentum has to be conserved in this discussion. So there needs to be some sort of momentum. But, in the center-of-mass frame, that's not required. So, in that frame, the momentum of all outgoing particles can be 0.

And that's how we start the discussion here. So, in this S prime frame-- here S prime is the center-of-mass frame-- the energy, the minimal energy required, is 2 times the mass of the proton times gamma. So here, two protons are colliding with the same velocity. And that's then equal to the energy after this process, c squared times the sum of the masses, the sum of the mass of the proton, the neutron, and the charged pion.

And then you just have to solve this for gamma to find gamma equal to 1.08 or beta in this frame of 0.37. Note, this is the gamma, relativistic gamma, or the velocity beta of the protons, two protons in the center-of-mass frame. So we're not quite there yet with our answer.

The answer then needs to be boosted back into the laboratory frame. And we have seen how we can do this for beta or velocities in general. We find beta in the laboratory frame is 2 times-- or just result, 0.37, over 1 plus 0.37 squared, which is 0.65.

That velocity, we can then take and calculate the gamma factor of the proton in the fixed-target experiment. All right, so we analyzed this situation in the center-of-mass frame and then did a Lorentz transformation by just looking at the velocity into the fixed-target frame.

So this means now, numerically, that the proton colliding with the proton at rest has a total energy of this one proton of gamma m0 c squared, which is 0.32 times 938 MeV over c-- MeV. And so that results in 1.238 GeV.

But we're interested in the kinetic energy. So the kinetic energy here is given by gamma minus 1 m0 c squared, which is 300 MeV. So we have to accelerate a proton to 300 MeV in order to be able to have this process to occur.

All right, very similar problem now, but here we want to produce anti-matter. So we have a process of proton plus proton into three protons and an antiproton. Charge is conserved. In the initial state, the charge was 2. In the final stage, the charge was plus 2 as well.

OK, this works very similar as in the previous problem. But what we want to do here is compare the fixed target with symmetric collisions.

OK, so, again, the question is, what is the minimal energy needed in order to produce antiprotons in proton-proton collisions? OK, so, exactly following the same procedure as before, in the center-of-mass energy, the energy is 2 times the mass of the protons times gamma times c squared. And that's 4 times the mass of the proton. OK, gamma prime, so the gamma factor in the center-of-mass frame is 2. Beta is 0.75.

And then we just do the very same thing again. We calculate the velocity in the fixed-target frame. And we find the velocity of beta of 0.96 and gamma of 3.57.

So, if we compare this now, we need a pair of 1 GeV-- remember, gamma minus 1 is the kinetic energy-- protons in a collider experiment or 2.57 GeV protons in a fixed-target experiment. OK, so you see that, in fixed-target experiment, in order to produce new particles, the energy has to be much larger, a factor of 2.5 here in this example, than a colliding experiment.

And that explains why we use collider experiments in order to test the energy frontier, in order to produce the largest possible energies. And the LHC is one example where we have proton-proton collisions in a circular ring where those protons are brought together in symmetrical collisions.