MARKUS KLUTE: Welcome back to 8.20, Special Relativity. In this section, we talk about spacetime diagrams. They turn out to be very useful tools to describe events or sequences of events, in particular when observed by multiple observers.

So what is a spacetime diagram? Here's an example. You have an x-coordinate and a t-coordinate for space and time. I plotted an event in blue here. And the world line of events. A world line is just a sequence of events as they occur. In this case, something seemed to be moving with constant velocity. The world line is just a continuous line of movement. The velocity of this event is delta x over delta t, which is 1 over the slope.

Let's have a look here. So the time axis is defined as those events which all occur at the same space, x equals 0, whereas the x-axis is defined as those events which all occur simultaneously at the same time. And then you can draw additional lines into the spacetime diagram where, for example, all times are equal to 1. You might want to add a unit. I omitted this here. Time might be given in seconds, in days, in hours, in years-- whatever you like-- similar to space in meters or light-years.

So lines here in green of the same time, meaning time is constant. All events on that line happen simultaneously, while in blue are those lines where events happened at the same location, so x is equal to constant some specific values.

Here in red, I add one additional caveat, which you're typically not aware or considering very much in diagrams is the role of tick marks. Here in this spacetime diagram, our tick marks are perpendicular to the axis. It's also OK or correct here to say that they are parallel to the second axis, and we'll come back to this point later on.

An example of a world line is simply drawing all events which correspond to me, right? Professor Klute is pacing in his office. You know, maybe on a line, just the x-coordinate is plotted here, time passes, and I'm just pacing the long-changing direction. Each time, each little segment is constant velocity. That's the world line of me from some time t for minus t to term time equals 40.

OK, so now our first concept question. Let's consider the set of world lines, 1, 2, 3, 4, and the question is, which of the objects which correspond to the world line is moving the slowest? Let's consider this for a second, and then we look at this. As the velocity is 1 over the slope, the object with the steepest slope, the largest value of the slope, is moving the slowest. And in this case, it's object number 2.

All right, now we want to actually make them useful. Yes, they can be used in order to describe certain event lines, but they're really useful when you describe events happening for different observer. So in this activity, I invite you to draw Bob's spacetime diagram into Alice's, and then as a second step, draw Alice's spacetime diagram into Bob's.

The situation is very similar to previous ones discussed in this lecture. Alice is stationary and Bob moving in this rocket with a velocity of half the speed of light, or a gamma factor of 1.2. All right, try. Go ahead. Try to show where is the time axis for Bob and where is the spatial coordinate for Bob.

OK, so the way to approach this is the following. We want to use Lorentz transformations in order to figure out what is the value of Bob's time axis and space axis for different values of Alice's spacetime diagram. So we start with drawing Alice's spacetime diagram. And then if you want to find the x-axis as seen by Bob, we have to set the time for Bob to equal 0 and then find the corresponding elements or tick marks on the axis.

So the first point we're going to find is tB equals 0 and xB equal 1. So with the Lorentz transformation, we find that xA 1, so this point corresponds in Alice's spacetime diagram to xA equal gamma equal 1.2 and tA equal gamma times v over C squared equal 0.6. So we can make this-- find this first point and plot it in our diagram. It's right here.

OK, and then we go move around and find the second point and the third point, and we do the same for the time axis, where xB is equal to 0 and tB equal 1, then corresponds to 4.6 in xA 1 and 1.2 in xA 2, where we find these points here. I failed to say that the origin of those two spacetime diagrams [INAUDIBLE]. OK?

So this is already it. So we found Bob's time axis, where xB is equal to 0, and Bob's x-axis, where tB is equal to 0. And I did draw those tick marks parallel to the second axis. So if I want to now find out the time axis for xB equal 1, I just have to follow along and draw a parallel in the picture here.

All right, so the second question then is, where Alice's axis in Bob's spacetime diagram? So the procedure is very similar as before. We draw Alice's axis-- sorry, we draw Bob's axis, and we find Alice's x-axis by setting tA equals 0, and then we find the number of points and connect those. And the time axis is found by setting xA equals 0 and finding them points for various values of time.

And you see here, this looks a little different than before. I just zoom in here a little bit. What you find specifically, because relative the direction of motion changes, the positive values of x are in positive value-- and negative-- so the positive values of the x-axis are in the negative time direction, while the negative values of x are in the positive time direction, so would be across down here.

So this needs a little bit to get used to, but you will later see when I draw in any of the two spacetime diagrams specific sequence of events, I can immediately read off how this event is perceived from Bob's and from Alice's perspective. And this makes our space diagram very, very useful tools.