[SQUEAKING]

[RUSTLING]

[CLICKING]

MARKUS KLUTE: Welcome back to 8.20 Special Relativity. So we're starting a new chapter. In this chapter, we talk about some aspects of special relativity, which are not crucially important to understand the concepts, but they help you to go a little bit deeper in your understanding. I hope this is going to be useful. So we want to talk about the algebra of known transformations. So we have seen that our gamma factor is 1 over square root 1 minus beta squared with beta equal the relativistic velocity v/c.

And thus, this we can rewrite it as gamma squared minus beta squared gamma squared is equal to 1, OK? So now, I would like you to recall hyperbolic functions sinh and cosh and cosh squared minus sinh squared is equal to 1. So the form here and here are pretty much the same. And something squared minus something else squared equal to 1, OK? Good.

So let's see how this looks like. As a reminder for us, the hyperbolic functions as defined as 1/2 e to the x power minus e to minus x and cosh equal to 1/2 e to x plus e to minus x. OK? The tangent is then defined as a ratio.

And you can plot those functions, and you can see the functional form as given in these two diagrams. Well, we want to come back to those two equations looking very much the same. So we can define now eta, the rapidity, as gamma equal to hyperbolic function cosh eta and beta gamma equals sinh eta. So basically, we have this rapidity, which is a measure on how much the system is boosted as being equal to this kind of hyperbolic angle, right?

You can then process again, where beta is equal to the tangents of this hyperbolic angle. And just remember that the slope in our space time diagram is 1 over the velocity. We find that angle again now is being called rapidity, OK? And just as a reminder, beta goes from minus 1 to 1, depending on the direction and the speed of it is less than the speed of light. And then eta goes from minus infinity to infinity.

OK, so then we can rewrite our Lorentz transformation. Instead of writing gamma and beta gamma and minus beta gamma and so on, we can write this through the hyperbolic angle. OK? So you should always ask why is this useful.

The first part is that when we add velocities, we found this complicated transformation where the new velocity is equal to the first velocity times the second velocity over 1 plus the product of the two velocities. And this is much easier now as we can just add the velocities. So the third velocity is equal to the first plus the second. This is much, much easier to actually calculate. And the proof of this is coming directly from the proof of those hyperbolic functions here.

The second part where this becomes useful is when you think about the angle in your space time diagram. How does this now compare to a normal rotation? So let's start here. So we have a normal rotation. We have a rotation at a angle, and our coordinate system just rotates by specific angle. Let's call it phi here.

And what we do now, we have a similar but a hyperbolic rotation in which the coordinate system in our space time diagram rotates. All right? In the normal rotational case, x squared plus y squared is invariant. And in our Lorentz transformation, c squared t squared minus x squared. All right?

If you then have a more general transformation, a rotation, and Lorentz transformation, you find x squared plus y squared plus z squared minus c squared t squared [INAUDIBLE]. OK, so we have just relabeled things, but now we can make use of everything we know about hyperbolic functions when we think about adding velocities. Because the rapidity-- the relative distance and speed between two reference frames is basically the angle of the hyperbolic angle.