Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
Term:	IAP 2021
Instructor:	Markus Klute

Midterm

January 21th, 2021

Rules:

This exam is "open book," which means you are permitted to use any materials handed out in class, your own notes from the course, the text books, and anything on the IAP21 8.20 canvas course website. The exam must be taken completely alone. Showing the exam or discussing it with anyone is forbidden. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes etc. You may not use Google or any other search engines for any reason. You may not use any shared documents. You may not consult with any other person regarding the exam. You may not check your exam answers with any person. You may not discuss any of the materials or concepts in 8.20 with any other person while taking the exam. In case of question, please consult the exam channel on the 8.20 slack workspace.

Task 1: Short Questions [16 points]

Indicate whether each statement is True or False. No justification is necessary.

(1) True / False - A photon can have momentum.
- True

(2) True / False - In an inertial frame with time coordinate t, the distance between any two objects cannot grow faster than c meters per second.
- False

(3) True / False - The product of a four-vector with itself is invariant under Lorentz transformation.
- True

(4) True / False - Consider the two events: "a plane takes off from Boston" and "six hours later, the plane lands in Los Angeles." These two events define a timelike interval.
- True

(5) True / False - For any pair of events A and B, there is always some inertial reference frame in which A and B occur in the same location.
- False

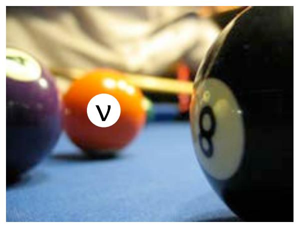
(6) True / False - The relativistic form of Newton's second law is $\vec{F} = \gamma m_0 \vec{a}$, where $\vec{a}$ is the acceleration, v is the speed, and $\gamma = (1 - v^2/c^2)^{-1/2}$.
- False

(7) True / False - If the density of an object is ρ in its rest mass frame, then when it is observed moving at speed v its density will be $\rho' = \gamma^3 \rho$.
- False

(8) True / False - The force and acceleration vectors are always parallel.
- False

Task 2: Neutrino Billiard [20 pts]

Consider a two body elastic collision between a neutrino and a nucleus at rest with mass m_A. For the purpose of this problem, assume the neutrino can be treated as a massless particle with initial energy E_ν. What is the *maximum* kinetic energy, K, that can be imparted to the nucleus during the collision?



- Let E'_ν be the energy of the scattered neutrino and K be the kinetic energy of the nucleus after the collision. Then by conservation of energy we have

$$E_\nu + m_A c^2 = E'_\nu + m_A c^2 + K$$
$$K = E_\nu - E'_\nu$$

But this is just the Compton scattering result we found in class. We will derive it again starting with conservation of 4-momentum.

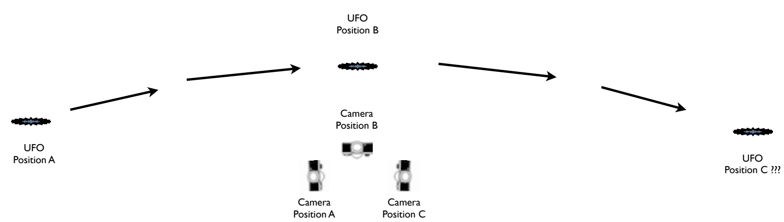
$$p_\nu + p_A - p'_\nu = p_A$$
$$(p_\nu + p_A - p'_\nu)^2 = (p_A)^2$$
$$-m_A^2 c^2 - 2E_\nu m_A + 2E'_\nu m_A + 2\frac{1 - E_\nu E'_\nu cos\theta}{c^2} = -m_A^2 c^2$$
$$\frac{1}{E'_\nu} - \frac{1}{E_\nu} = \frac{1 - cos\theta}{m_A c^2}$$
$$K = E_\nu - E'_\nu = E_\nu \frac{1 - cos\theta}{m_A c^2 / E_\nu + 1 - cos\theta}$$

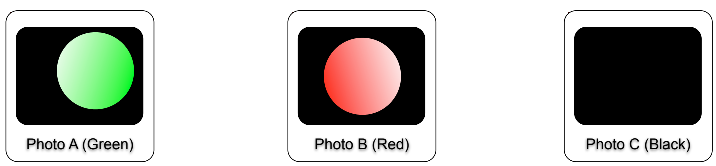
For maximum K we need $cos\theta = -1$. As a result

$$K = \frac{2E_\nu^2}{m_A c^2 + 2 \cdot E_\nu}$$

Task 3: UFO Flyby [20 points]

A U.F.O. is spotted across the dark night sky as it approaches a telescope located on the ground. The telescope is equipped with a camera which operates in the visible spectrum and that can attempt to track fast-moving aircraft. The camera only manages to take three pictures in the visible spectrum. The first is taken when the object is very far away and it appears green ($\lambda = 550$ nm). The second is taken when the object is directly overhead and it appears red ($\lambda = 730$ nm). The third is taken as the object flies away, but unfortunately the photograph appears blank.





Assume that the UFO is moving at a constant velocity β with respect to the telescope.

(a) What is the approximate speed of the object?

(b) Why was the third photograph blank?

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(a) To find the speed of the object, we can make use of the relativistic Doppler shift effect to determine the relative velocity of the incoming/outgoing object (even without knowing the original color of the object). From the frequency Doppler shift formula, we find for the three camera positions:

$$\lambda_A = \lambda_0\gamma(1-\beta)$$
$$\lambda_B = \lambda_0\gamma$$
$$\lambda_C = \lambda_0\gamma(1+\beta)$$

Hence

$$\lambda_A/\lambda_B = (1-\beta)$$
$$\beta = 1 - \frac{\lambda_A}{\lambda_B}$$
$$\beta = 1 - \frac{550\text{ nm}}{720\text{ nm}} \simeq 0.25$$

(b) The third photograph is blank because, as it recedes away, it slips into the infrared spectrum (and is no longer visible).

$$\lambda_C/\lambda_A = \frac{(1+\beta)}{1-\beta}$$
$$\lambda_C/\lambda_A = \frac{(1+0.25}{1-0.25}$$
$$\lambda_C = (500\text{ nm}) \cdot \frac{9}{5} = 833\text{ nm} \quad \text{(infrared)}$$

Task 4: Twin Paradox [22 points]

Alice and Bob are having one last relativistic adventure, or at least Alice is. They both start at $x_A = x_B = 0$, where they synchronize their watches. At $t_A = t_B = 0$, Alice takes off in a spaceship at $v = \frac{3}{5}c$. After $t_B = 10$ yr, Bob observes Alice's spaceship to turn around instantaneously and begin traveling towards him at $v = \frac{3}{5}c$. Finally, at $t_B = 20$ yr, Alice and Bob are reunited and compare their watches.

(a) Draw the spacetime diagram for Alice's trajectory in Bob's rest frame.

(b) Draw the axes for Alice's reference frame(s) in Bob's rest frame.

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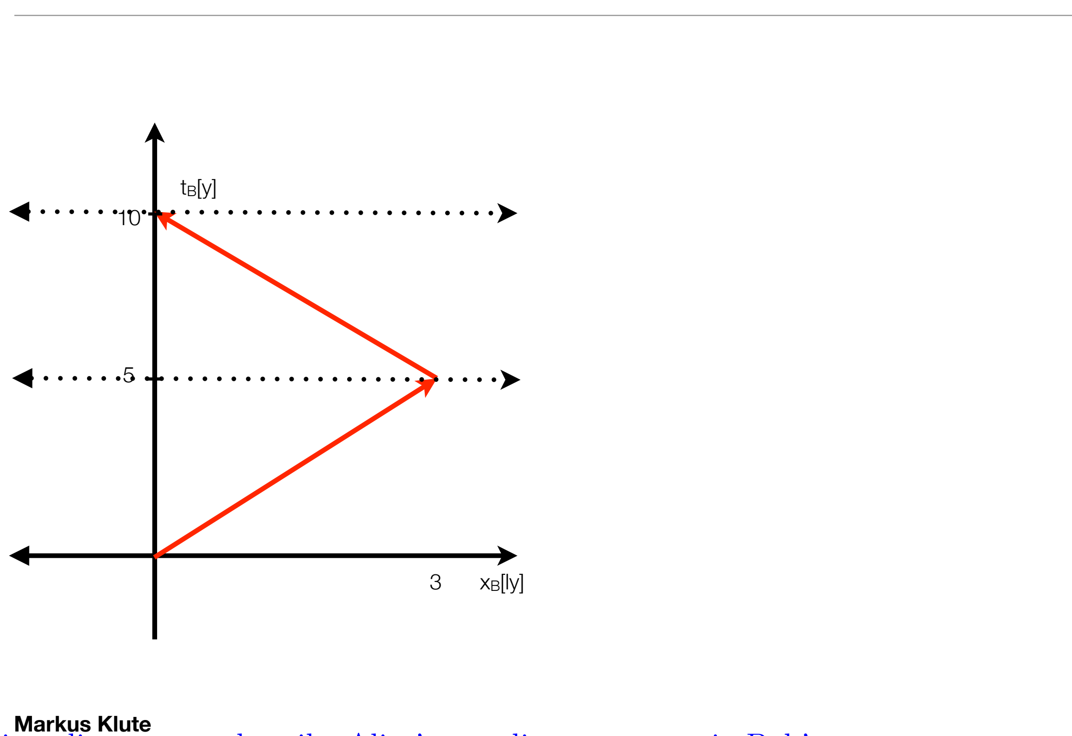
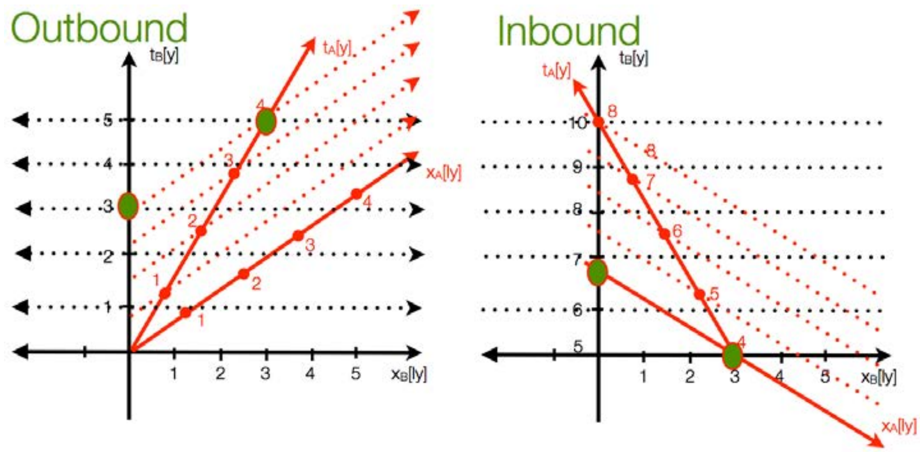


(b) You need two spacetime diagrams to describe Alice's coordinate systems in Bob's rest frame – one while she's moving away and one while she's moving towards him.



(c) Either because Alice is leaving an inertial reference frame by accelerating when she turns around, or because she has two different inertial reference frames throughout her journey, she's the one that's actually younger.

Task 5: The Barn Paradox [22 points]

A runner is holding a pole that is 12 meters long in its rest frame. He is running towards a barn that is 10 meters long in its rest frame. The barn has two doors: a front door that is initially open, and a back door that is initially closed.

The runner has a speed $v = \sqrt{3}/2c$ ($\gamma = 2$). He has two friends, one at each door of the barn. The friend at the front door closes the front door as soon as the pole is completely past. The friend at the back door opens the back door just before the pole would hit it.

(a) According to the two friends, what is the length of the pole?

- Because the pole is moving with respect to the friends' reference frame, its length is contracted such that

$$L_f = \frac{1}{\gamma} L_r = \frac{12\text{ m}}{2} = \boxed{6\text{ m.}}$$

(b) According to the runner, what is the length of the barn?

- Because the barn is moving with respect to the runner's reference frame, its length is contracted such that

$$L_r = \frac{1}{\gamma} L_f = \frac{10\text{ m}}{2} = \boxed{5\text{ m.}}$$

(c) According to the two friends, are both doors ever closed at the same time? Briefly explain.

- Yes, the two friends do see the two doors closed at the same time because according to them the pole is short enough to fit in the barn.

(d) According to the runner, are both doors ever closed at the same time? Briefly explain.

- No, because he thinks the pole is more than twice the length of the barn.

(e) If there is a difference between the two viewpoints, explain it.

- Events at different locations that are simultaneous in one reference frame are not simultaneous in another.

8.20 Introduction to Special Relativity
IAP 2021