Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
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Instructor:	Markus Klute

Problem Set 4

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Note: In most problems, I will work with the convention that a 3-vector is denoted by an arrow ($\vec{p}$), while a 4-vector would just be denoted as p (I have dropped the subscript term, p^μ, as this is the convention). In particular physics, we often set the speed of light to the unit-less value of 1 (c = 1). Masses are often referred to in terms of energy (i.e. 511 keV, 938 MeV, etc...)

Problem 1: Acceleration in Special Relativity [25 pts]

In class we determined that the momentum of a particle traveling at velocity $\vec{u}$ with respect to an observer is given by $\vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}$.

(a) Find the force $\vec{F}$ by taking the derivative with respect to ordinary time.

(b) It is possible to also define a 4-vector for acceleration, just like we did for 4-velocity, by taking again the time derivative with respect to proper time

$$\alpha^\mu = \frac{d\eta^\mu}{d\tau} = \frac{d^2x^\mu}{d\tau^2}$$

Find the components of α^μ.

(c) Express those components in terms of the force term you found in part (a).

Problem 2: π^0 Decay [25 pts]

The π^0 is a heavy meson with a mass of 135 MeV/c^2 that decays almost immediately to two back-to-back photons (with a lifetime of $\tau = 8.4 \times 10^{-17}$ s).

(a) What are the energies of the two photons emitted in the center-of-mass frame of the π^0 when it decays?

(b) Suppose one of the two photons makes an angle θ with respect to the x-axis in the center of mass frame. What is the minimum energy the π^0 must have in order for both photons to be boosted in the forward direction (i.e. make an angle less than 90° from the positive x-axis)? This is convenient if your detector doesn't fully encompass the region surrounding your pion.

(c) Suppose with your detector (read as, lab frame) you measure both photons and each makes a $\pm 45°$ angle with respect to the beam axis. From this information, tell me how far the π^0 moved from when it was created to when it decayed.

Problem 3: Review: Mendelstam Variables [25 pts]

High energy physicists try as best they can to express various quantities (energy, momentum, cross-sections, etc.) in terms of invariant quantities. This is not mere aesthetics; it is far easier to make calculations if those calculations are independent of what frame one is working in. One such tool are Mendelstam variables, which describe the energy-momentum exchange when 2 particles collide with one another.

Consider the (inelastic) collision of two particles (1 and 2) to yield two different particles (3 and 4), each with a different mass $m_{i=1,2,3,4}$. The Mendelstam variables are defined as follows:

$$s \equiv (p_1 + p_2)^2/c^2$$
$$t \equiv (p_1 - p_3)^2/c^2$$
$$u \equiv (p_1 - p_4)^2/c^2$$

(a) Calculate the quantity $s + t + u$.

(b) Find the lab-frame energy of particle 1 in terms of Mendelstam variables (Suppose we are working with a fixed-target experiment, where lab frame implies particle 2 is at rest.)

(c) Finally, find the total center-of-mass energy ($E_1 + E_2$) in terms of Mendelstam variables.

Problem 4: Collider versus Linac [25 pts]

Suppose you were determined to discover a new particle (say, the Higgs) that required a very high energy center-of-mass energy, $\sqrt{s}$ to create[1]. You decide you will create this elusive particle by slamming two identical particles of mass m (say, two protons) against each other. You have two choices on how to build your machine. You can build either (a) a collider, which slams the two particles in a head-to-head collision or (b) a linac (linear accelerator) which slams one particle against a fixed stationary target of material.

(a) For a given kinetic energy K (which is either given solely to the proton in the linac or split evenly among the two protons in the collider) which option is the better choice to reach your targeted center-of-mass energy?

(b) For what other reasons might you chose the other, less energetic, option?

Figure 1: Photograph of the Fermi National Accelerator Facility. One can see both fixed target beamlines (linacs) and the main proton-anti-proton collider.

[1] If you are confused as to what $\sqrt{s}$ is, look at Task 3 in this problem set.