Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
Term:	IAP 2021
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Midterm

January 14th, 2021

Rules:

This exam is “open book,” which means you are permitted to use any materials handed out in class, your own notes from the course, the text books, and anything on the IAP21 8.20 canvas course website. The exam must be taken completely alone. Showing the exam or discussing it with anyone is forbidden. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes etc. You may not use Google or any other search engines for any reason. You may not use any shared documents. You may not consult with any other person regarding the exam. You may not check your exam answers with any person. You may not discuss any of the materials or concepts in 8.20 with any other person while taking the exam. In case of question, please consult the exam channel on the 8.20 slack workspace.

Task 1: Short Questions [10 points]

Answer the following questions *briefly*. No calculations are needed. In several cases, a one word answer will suffice.

(a) What property of a reference frame makes it an inertial frame?

- An inertial frame is a reference frame in which Newton's first law holds: In absence of external forces particles travel in a straight line with a constant velocity.($\vec{a} = 0$).

(b) Is the following statement true or false: "Prior to Einstein, nobody had noticed that the laws of Newtonian mechanics satisfy a principle of relativity"

- False

(c) Explain why the Michelson-Morley experiment never yielded any fringe shifts, regardless of the orientation of the experiment, the time of day, or the time of year.

- Speed of light is the same in all frames. There is no ether.

(d) Is the following statement true or false: "Events A and B occur at the same place in an inertial frame, with A happening before B. It follows that A occurs before B in any other inertial frame."

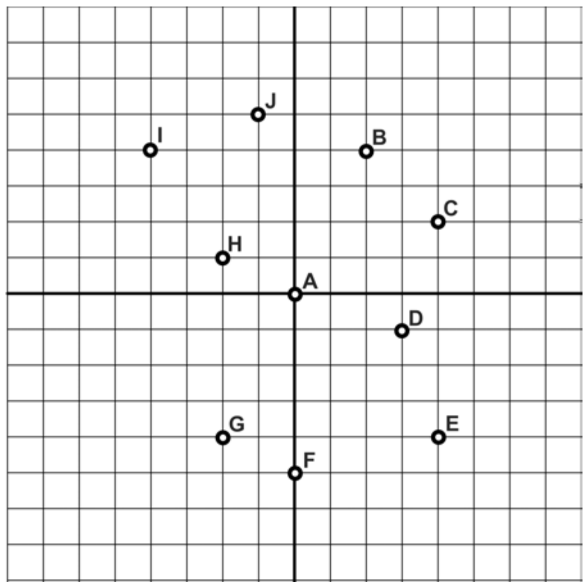
- True

(e) Critique the statement: "The speed of the light emitted by my laser is the same in all frames. So is its frequency".

- Frequency is not an invariant quantity

Name:

Task 2: Space Time Diagram [15 points]



In the space-time (ct, x) diagram above, each horizontal unit is 1 lightyear and each vertical unit is 1 year.

(a) Give one pair of events that are simultaneous in this frame.

- I and B, G and E

(b) Give one pair of events that take place at the same position in this frame.

- C and E, H and G, A and F

(c) Give one pair of events that have a light-like separation.

- A and E, A and I, I and E

(d) List all events in that could be caused by A.

- B, J, I

(e) List all events that could have caused A.

- G, F, E

Task 3: Signaling a Rocket[15 points]

A rocket is moving with the speed v along the x-axis, as seen by observers at rest in frame S. The rocket passes $x = 0$ at $t = 0$. At time $t = T$ a light signal is emitted from $x = 0$ in the direction of the rocket.

(a) At what time t and position x does the light signal reach the rocket?

- $$t = \frac{cT}{c - v}$$
  $$x = \frac{cvT}{c - v}$$

(b) According to the rocket pilot, how much time, T', has elapsed between the time when she passed the origin of S and the time when she receives the light signal? Express T' in terms of T, v, and c.

- According to Lorent'z transformations $T' = \gamma(t - \frac{vx}{c^2})$. For the event coordinates found in part (a) we have

  $$T' = \frac{\gamma cT}{c - v}(1 - \frac{v^2}{c^2}).$$

  This can be simplified further using the definition of $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ and $\beta = v/c$.

  $$T' = T(\frac{1 + \beta}{1 - \beta})^{\frac{1}{2}}.$$

Task 4: To Our Children's Children's Children [20 points]

The star Gliese 832 is located 16 lightyears away from the Earth. Suppose we send a spacecraft to investigate the recently discovered exoplanet in that star system with velocity $v = \frac{4}{5}c$ and $\gamma = \frac{5}{3}$.

(a) According to the occupants of the spacecraft, how long does the trip take?

- According to someone in the Earth's rest frame, traveling 16 ly at a velocity of $v = \frac{4}{5}c$ m/s will take time

$$\Delta t_E = \frac{16 \text{ ly}}{(4/5) \text{ ly/yr}} = \left(\frac{5}{4} \text{ yr/ly}\right)(16 \text{ ly}) = 20 \text{ yr}.$$

Someone on the spacecraft moving at a relative velocity v will experience time dilation compared to the observer on earth and measure an elapsed time of

$$\Delta t_{SC} = \frac{1}{\gamma}\Delta t_E = \frac{3}{5}(20 \text{ yr}) = \boxed{12 \text{ yr.}}$$

This answer makes "intuitive" sense because moving clocks run slow so the moving clock on the spacecraft will measure less time than an observer back on earth.

(b) According to the occupants of the spacecraft, how far did they travel?

- According to a stationary observer in Earth's rest frame, the spacecraft traveled 16 ly. However, due to length contraction the occupants of the spacecraft measure the distance to be

$$\Delta L_{SC} = \frac{1}{\gamma}\Delta L_E = \frac{3}{5}(16 \text{ ly}) = \frac{48}{5} = \boxed{9.6 \text{ ly.}}$$

You can double-check this answer from (a) and no further relativity by calculating

$$\Delta L_{SC} = v\Delta t_{SC} = (4/5 \text{ ly/yr})(12 \text{ yr}) = 9.6 \text{ ly}.$$

(c) Once they arrive, they send a light-based signal back to Earth. According to the people on Earth, how much time passed between the spacecraft leaving and the signal being recieved?

- According to the Earth-based observer, it took them 20 yr to get to Gliese 832, and then 16 yr for the light to return. So $\boxed{36 \text{ yr}}$ total.

Task 5: Addition of Velocities [20 points]

An observer on a spaceship observes a speeding bullet, travelling to the right with speed u' and a second bullet speeding upwards with the speed u''. An observer on earth observes, in her reference frame, the spaceship moving to the left with speed v. Use Lorentz transformations to derive an expression for the speed of the bullets in the frame of the earthbound observer.

- Let's call the spaceship frame S and earth frame $\overline{S}$.

$$u' = \frac{dx}{dt}$$
$$u'' = \frac{dy}{dt}$$
$$\overline{u'} = \frac{d\overline{x}}{d\overline{t}}$$
$$\overline{u''} = \frac{d\overline{y}}{d\overline{t}}$$

According to Lorent'z transformations we have

$$d\overline{x} = \gamma(dx - vdt)$$
$$d\overline{t} = \gamma(dt - \frac{v}{c^2}dx)$$
$$d\overline{y} = dy.$$

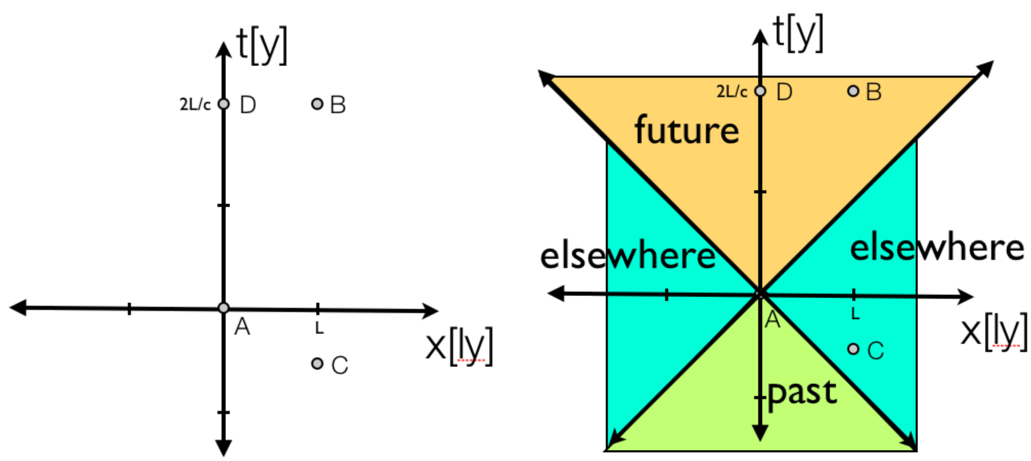
Putting everything together we obtain the speed of the bullets in the frame of the earthbound observer.

$$\overline{u'} = \frac{u' - v}{1 - \frac{vu'}{c^2}}$$
$$\overline{u''} = \frac{u''}{\gamma}$$

Task 6: Cause and Effects [20 points]

In an inertial frame of reference, four events A, B, C, and D have the following coordinates:
A: $x_A = 0$, $t_A = 0$
B: $x_B = L$, $t_B = 2L/c$
C: $x_C = L$, $t_C = -L/2c$
D: $x_D = 0$, $t_D = 2L/c$
Here, $L > 0$ and you can ignore the y- and z-directions.

(a) Draw a spacetime diagram, showing events A, B, and C. Draw and label the regions of the diagram which constitue the past, future, and elsewhere relative to event A.



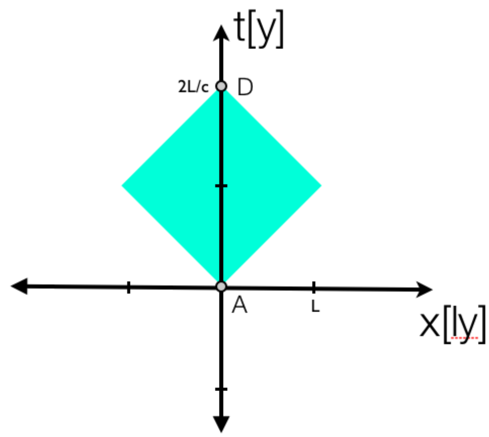
(b) Could event A have caused event B?
- Yes

(c) Could event A have caused event C?
- No

(d) Could event C have caused event A?
- No

(e) Could event C have caused event B?
- Yes

(f) Now draw a spacetime diagram showing events A and D only. Given that your world line passed through A and D and given that your speed during the intervening time never exceeds c, draw and shade the region of spacetime you might have visited during the time between A and D.



(g) Draw a new spacetime diagram, again showing A and D only. This time, draw and shade the region of spacetime consisting of events which could not have affected you at A, but could have affected you at D.