[SQUEAKING]

[RUSTLING]

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PROFESSOR: Welcome back to 8.20. In this section and the following ones, we talk about paradoxes in special relativity.

A paradox is something which is absurd or self-contradictory. So we have statements which don't really make any sense when put together.

The pole in the barn paradox is rather interesting. And we will analyze this, and at the end of the discussion, we hopefully agree that there is no paradox here. It's a pseudo paradox.

So the situation is as follows. We have Alice. She has a pole. The pole is 10 meters long in her reference frame.

And Bob is very proud of his New England barn which is, in his reference frame, 8 meters long. Alice, however, is moving with a velocity of 0.6 times the speed of light, which gives us a gamma factor of 1.25.

Does the pole fit into the barn is the question, or not? So stop here and think about this for a second, and we will continue with an analysis of this. So here is the analysis for Alice's frame and the analysis for Bob's frame.

For Alice, the barn in her reference frame is Lorentz contracted. It's 6.4 meters long, but her pole is 10 meters long. So we should clearly answer this question by saying it doesn't fit.

In Bob's frame, the barn is 8 meters long, and the pole is Lorentz contracted-- also 8 meters long. So Bob will say, yeah, it fits-- it just barely fits. They're exactly the same size, so yes, it all fits into the barn.

And here is where you might think this is an absurd statement. They cannot be both right. We will see they can. They can both be right. They just disagreed on the fact that events happen simultaneously.

What are the crucial events here? When does the barn hit the end-- does the pole hit the end of the barn, and when does the back of the pole hit the front of the barn? Those are the two things we have to study in detail.

But let's get to it. How can they, or why can they disagree? So the idea is that you draw space-time diagrams for the pole in the barn, and show that there's no paradox by using the world lines of the pole.

Before we do this, we're going to analyze this a little bit more. So assume that the front of the pole enters the barn at time equals 0 for both Bob and Alice.

Then Bob observes the pole entering his barn, and it takes 44.4 nanoseconds-- 8 meters divided by 0.6 times the speed of light-- for the front of the pole to reach the back of the barn, and the back of the pole to reach the front of the barn. So after 44 nanoseconds, in Bob's reference frame, the pole is in the barn.

Alice, however, sees the barn Lorentz contracted. It's 6.4 meters long. She moves this 0.6 times the speed of light.

So for her, she reaches the back after 35.6 nanoseconds, in which case, Bob's clock only shows 28.4 nanoseconds, because Alice's clock time is Lorentz contracted. So we can clearly conclude here that that's not enough time for Bob such that the pole actually entered the barn for the full length. So the back of the pole is still outside.

So we want to consider three different events. The first event is after 44 nanoseconds, and in the space of 8 meters in Bob's reference frame. For Alice-- this is the situation we just analyzed-- 36.6-- 35.6 nanoseconds passed. And in her reference frame, the front of the pole is at 0 meters.

The second event is then the other side of the barn in Bob's reference frame after 44.4 nanoseconds 0 meters. He sees-- or she sees that 55 nanoseconds have passed. We use Lorentz transformation here, but the position is minus 10 meters.

And the last point is 28.49 nanoseconds and 0 meters. That is the observation when Alice sees the end of the-- front of the barn-- the front of the pole at the end of the barn.

That translates into Alice's frame a 35.6 nanoseconds, and minus 6.4 meters. So the minus 6.4 meters tells you very clearly what we just already said. The back of the pole is still outside.

So that's the quantitative or numerical kind of evaluation. And we can also show the very same thing in the space-time diagram. So we show the space-time diagram here, and this is Bob's reference frame. So the pole just touched the front of his barn, and the barn is located at 8 meters-- the end of the barn is located 8 meters. The front of the barn is located at 0 meters.

After 44 nanoseconds, there's event number one and event number two. The pole is fully in the barn. But we can also show the pole in event number three.

So one-- where is the end of the pole? We look at this diagram here. Where is the end of the pole when the front of the pole hits the end of the barn?

You see clearly there's a piece sticking out. We saw that there's-- in this event here, 6.6 meters in Alice's frame still seeking out.

So we see that event number three is located here, and not all of the pole is actually contained within the. So Bob and Alice disagree on whether the front and the back of the pole are in the barn simultaneously.

That's where the situation becomes contradictory. They don't agree that two events which happened at the same time in their reference frame-- in Bob's reference frame occurs at the same time in Alice's reference.