MARKUS KLUTE: Welcome back to 8.20, Special Relativity. In this last example of relativistic kinematics, we want to investigate scattering-- in this specific case, a scattering of a photon on an electron at rest. So we have as an initial state a photon, an electron at rest, and then the photon is scattered and we also observe a scattered electron.

There's one important piece of physics here, which we add without further explanation, which is the Planck-Einstein relation, which relates the energy of the photon to the frequency of the photon or the wavelength. This is fundamentally important in quantum physics, and can be explained or tested with the photoelectric effect for which Einstein received the Nobel Prize.

So what we want to do here is find the wavelength shift, so delta lambda, which is the wavelength of the incoming photon minus the wavelength of the outgoing photon, as a function of the scattering angle zeta, as shown in this picture here. OK, so again, this is an activity I want you to work on and try to find out this. The algebra here is not trivial, but knowing how to set up a problem like this is important. So let's try.

So the way to set this up is to write this four vector relation, or you could just simply write down energy conservation and momentum conservation. So you have an initial state, the before, and the final state, the after, where you simply add the four vectors of the initial electron and photon and set this equal to the scattered electron and scattered photon.

Now, we are interested in a quantity delta lambda, which is related to the change in energy of the photon. So therefore, it brings the four vector of the four scattered photon over here to this side, and builds a square, which allows us then to use our invariant information in the scattering process.

When we explore the squared here, we find the photon four vectors squared for the scattered and the unscattered photon, minus 2 times the product of the two four vectors. Now, the mass of the photon is 0, and then hence the invariant mass is 0, too, so this invariant four vector is 0. So this cancels and this cancels. And then we know that the mass of the electron is the mass of the electron, the initial momentum of the electron is 0, and we just for the further, not to get confused, we said C equal to 1.

So then we just go through a sequence of algebra here, making use of information that those guys here are simply the mass of the electron. And we move things around a little bit and then find this equation here, which relates the energies of the two photons [INAUDIBLE] the scattering angle, which we get from the scattered product of the [? three ?] momentum of the photon to the change in electron energy, which is the energy of the electron minus the mass. OK.

And then we start using the Einstein relation here. And again, a little bit of algebra then brings us to delta lambda equal h over me times 1 minus cosine theta. So this relates the shift in wavelengths to the scattering angle of the photon. Important. If you want to recall this, the most important part through this problem is setting up this first equation here, which relates the energy and the momentum, or the four vector of those particles, before and after the collision.

And again, then it takes a little bit of practice. But the way to approach most of this problem is to make use of the invariant four vector squared, or the invariant mass of the objects involved, if we know the masses of the object involved. OK.