MARKUS KLUTE: Welcome back to 8.20, special relativity. In this section, we want to discuss Lorentz transformation. Or, in other words, given an event observed by Bob, we want to express that event as observed by Alice. We want to find the translation between the observations in Bob's reference frames to the observation in Alice's reference frames.

We have already done this for the classic case as Galilean transformation. Now, we want to do this in the framework of special relativity.

In order to simplify the discussion, we don't worry about the y- and z-component here. Those dimensions can be neglected if we assume that the relative motion between the two reference frames only in x-direction.

We also know from the previous discussion that you can use the invariant interval. ct squared minus x squared is the same observed in Bob's and in Alice's reference frame. We'll make use of this fact.

And, lastly, we can assume that this transformation has to be linear. Why? Because we transform something like a measurement of distance into a measurement of distance. It has to be linear. If not, we find something like a length squared or the same for time. And we might end up on time squared if we don't do this correctly.

All right, so we can write this down as a linear equation, which is a multiplication of a matrix with a vector, ct, x, into a vector, ct [? x ?] [? prime, ?] x prime. OK, so the goal here now is to find the parameters or the coefficients of this matrix, OK?

I invite you to stop the video here and try to work it out. It's an interesting exercise. It tests your algebra knowledge. There's not much physics in here, but it's still useful to go along and try to work this out.

So the first thing we want to do is assume that the origins coincide at t equals 0. And then we can follow along the trajectory of the origin of S prime in the S frame. So this is just ct, vt. OK, great. This already gives us a constraint on the coefficients a1,0 over a1,1, which is equal to minus v/c, OK?

And then we can use the invariant interval, which is another constraint. And we can use this to obtain the set of equations here. I will not read this for you. And that's already enough in order to solve the set of equations.

So, if you do this and follow along, you find answers for all four coefficients given gamma and beta as we defined them before. This then simplifies to our Lorentz transformation. So the only thing we did here is we simplified a little bit. We assumed that this is a linear transformation. We used the invariant interval in order to set the constraints. And we find Lorentz transformation.

If I summarize this, we find this matrix here with coefficients gamma, minus gamma beta, minus gamma beta, and gamma. Great. Or, if you want, you can write this as an equation for the spatial component and the time component.

So does this make sense? There's always a chance that we make a mistake in this kind of calculation. So we want to make sure that the answers we developed in previous sections actually are reflected by this transformation.

So let's go one by one. The first thing we can do is check units. If we do that, we see that this first equation here is of unit meter, and then we can analyze the second part of the equation.

OK, so gamma is unitless. x is of unit meter. And then we have beta ct. Beta is unitless. c is meter per second times second, also of unit meter. So this checks out.

The second equation is very similar. c times t is of unit meter. Meter per second times second is of unit meter. Gamma is unitless. Beta is unitless. And then we have an x, unit [? meter, ?] plus ct, c, meter per second times second, also meter.

So this checks out. So this is great. At least we find that we have a linear transformation by design, and the units work out.

So now we can see, what happens now if we use this for velocities which are much, much smaller than the speed of light? In this case, gamma is equal to 0, and beta is very close to 0. If we put this in our equations, you find x prime is equal x minus vt. And t prime is equal to t.

OK, this checks out because this is our Galilean transformation. So, for systems which move relative with very low difference in velocities, we can use Galilean transformation as an approximation of Lorentz transformation.

OK, at a third part, now we can investigate a little bit further. For example, what happens now to a distance, just a measure of distance or a measure of length, which we obtain by making this measurement simultaneously at t2 equal to t2? We find delta x prime is equal to gamma delta x. All right, that's length contraction.

If we do the same thing for delta t, for doing the measurement of time at x equals-- x2 equals x1, we find time dilation. All right, this is exactly what we expect.

And then we can look at two events which happen at the same time in frame S and see what happens to the time, as measured in system S prime. Delta t prime is equal to gamma delta t. Well, in this example, we set this to 0.

And then we have the second term, which is minus beta over c gamma delta x. So we find that, while this event happened simultaneously in our frame S1-- or in S, it does not happen simultaneously in our frame S prime. There's an extra term, which is not 0 unless you actually measure at the very same point, l is equal to 0.

So this is the relativity of simultaneity. Again, this checks out. And I think we're good with our Lorentz transformation.